# A.P. Calculus BC First Semester Exam
# Calculators Allowed

Two Hours

Number of Questions—10

Each of the ten questions is worth 10 points. The problem whose solution you write counted again, so that the maximum possible points earned is 110 (taken out of 100). There is no penalty for guessing.

- SHOW ALL YOUR WORK. You will be graded on the methods you use as well as the accuracy of your answers. Correct answers without supporting work may not receive full credit.
- Write all work for each problem in the space provided. Be sure to write clearly and legibly. Erased or crossed out work will not be graded.
- Justifications require that you give mathematical (non-calculator) reasons and that you clearly identify functions, graphs, tables, or other objects that you use.
- You are permitted to use your calculator to solve an equation or graph a function without showing work. However, you must clearly indicate the setup of your problem.
- Your work must be expressed in mathematical notation rather than calculator syntax. For example, $\int_1^5 x^2\,dx$ may not be written as `fnInt(X^2,X,1,5)`.
- Unless otherwise specified, answers (numeric or algebraic) need not be simplified. If your answer is given as a decimal approximation, it should be correct to three places after the decimal point.

*Directions:* Solve each of the following problems on the answer sheets, not on this sheet! Write your name on the problem whose solution you write.

**Good Luck!**

1. Let $y = f(x)$ be the continuous function that satisfies the equation $x^4 - 5x^2y^2 + 4y^4 = 0$ and whose graph contains the points $(2,1)$ and $(-2,-2)$. Let $\ell$ be the line tangent to the graph of $f$ at $x = 2$.

   (a) Find an expression for $y'$.

   (b) Write an equation for line $\ell$.

(c) Give the coordinates of a point that is on the graph of $f$ but is not on line $\ell$.

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(d) Give the coordinates of a point that is on line $\ell$ but is not on the graph of $f$.

[1980 AB/BC]

2. A particle moves along the $x$-axis so that its acceleration at any time $t$ is given by $a(t) = 6t - 18$. At time $t = 0$ the velocity of the particle is $v(0) = 24$, and at time $t = 1$ its position is $x(1) = 20$.

   (a) Write an expression for the velocity $v(t)$ of the particle at any time $t$.

   (b) For what values of $t$ is the particle at rest?

(c) Write an expression for the position of the particle at any time $t$.

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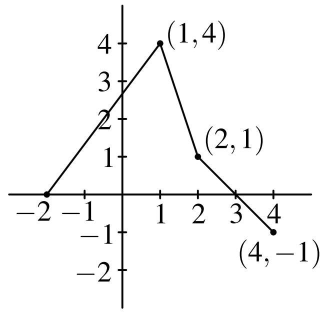
(d) Find the total distance traveled by the particle from $t = 1$ to $t = 3$.

[1987 AB]

3. The graph of the function $f$, consisting of three line segments, is given. Let $g(x) = \int_1^x f(t)\, dt$.




(a) Compute $g(4)$ and $g(-2)$.

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(b) Find the instantaneous rate of change of $g$, with respect to $x$, at $x = 1$.

(c) Find the absolute minimum value of $g$ on the closed interval $[-2,4]$. Justify your answer.

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(d) The second derivative of $g$ is not defined at $x = 1$ and $x = 2$. Which of these values are $x$-coordinates of points of inflection of the graph of $g$? Justify your answer.

[1999 AB/BC]

4. The temperature outside a house during a 24-hour period is given by

$$F(t) = 80 - 10\cos\left(\frac{\pi t}{12}\right), \quad 0 \le t \le 24,$$

where $F(t)$ is measured in degrees Fahrenheit and $t$ is measured in hours.

(a) Find the average temperature to the nearest degree Fahrenheit, between $t = 6$ and $t = 14$.

(b) An air conditioner cools the house whenever the outside temperature was at or above 78 degrees Fahrenheit. For what values of $t$ was the air conditioner cooling the house?

(c) The cost of cooling the house accumulates at the rate of \$0.05 per hour for each degree the outside temperature exceeds 78 degrees Fahrenheit. What was the total cost, to the nearest cent, to cool the house for this 24-hour period?

[1998 AB/BC]

5. A particle moves along the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 3t^2 - 2t - 1$. The position $x(t)$ is 5 for $t = 2$.

   (a) Write a polynomial expression for the position of the particle at any time $t \geq 0$.

   (b) For what values of $t$, $0 \leq t \leq 3$, is the particle's instantaneous velocity the same as its average velocity on the closed interval $[0, 3]$?

(c) Find the total distance traveled by the particle from time $t = 0$ until time $t = 3$.

[1997 AB]

6. For time $t \geq 0$ hours, let $r(t) = 120\left(1 - e^{-10t^2}\right)$ represent the speed, in kilometers per hour, at which a car travels along a straight road. The number of liters of gasoline used by the car to travel $x$ kilometers is modeled by $g(x) = 0.05x\left(1 - e^{-x/2}\right)$.

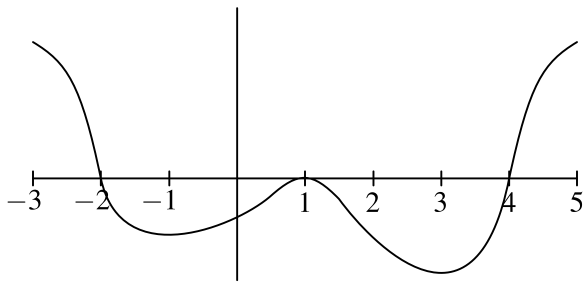
   (a) How many kilometers does the car travel during the first 2 hours?

   (b) Find the rate of change with respect to time of the number of liters of gasoline used by the car when $t = 2$ hours. Indicate units of measure.

(c) How many liters of gasoline have been used by the car when it reaches a speed of 80 kilometers per hour?

[2008 AB/BC]

7. The figure below shows the graph of $f'$, the derivative of a function $f$. The domain of $f$ is the set of all real numbers $x$ such that $-3 < x < 5$.




(a) For what values of $x$ does $f$ have a relative maximum? Why?

(b) For what values of $x$ does $f$ have a relative minimum? Why?

(c) On what intervals is the graph of $f$ concave upward? Use $f'$ to justify your answer.

(d) Suppose that $f(1) = 0$. Draw a sketch of $f$ that shows the general shape of the graph on the open interval $0 < x < 2$.

[1996 AB]

8. The rate of fuel consumption, in gallons per minute, recorded during an airplane flight is given by a twice-differentiable and strictly increasing function $R$ of time $t$. A table of selected values of $R(t)$, for the time interval $0 \leq t \leq 90$ minutes, is shown below.

| $t$ (minutes) | $R(t)$ (gallons per minute) |
|---|---|
| 0 | 20 |
| 30 | 30 |
| 40 | 40 |
| 50 | 55 |
| 70 | 65 |
| 90 | 70 |

(a) Use data from the table to find an approximation for $R'(45)$. Show the computations that lead to your answer. Indicate units of measure.

(b) The rate of fuel consumption is increasing fastest at time $t = 45$ minutes. What is the value of $R''(45)$? Explain your reasoning.

(c) Approximate the value of $\int_0^{90} R(t)\,dt$ using a left Riemann sum with the five subintervals indicated by the data in the table. Is this numerical approximation less than the value of $\int_0^{90} R(t)\,dt$? Explain your reasoning.

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(d) For $0 < b \leq 90$ minutes, explain the meaning of $\int_0^{b} R(t)\,dt$ in terms of fuel consumption for the plane. Explain the meaning of $\frac{1}{b}\int_0^{90} R(t)\,dt$ in terms of fuel consumption for the plane. Indicate units of measure in both answers.

[2003 AB]

9. Let $f$ be a function such that $f''(x) = 6x + 8$.

   (a) Find $f(x)$ if the graph of $f$ is tangent to the line $3x - y = 2$ at the point $(0, -2)$.

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   (b) Find the average value of $f(x)$ on the closed interval $[-1, 1]$.

[1989 AB/BC]

10. The rate at which water flows out of a pipe, in gallons per hour, is given by a differentiable function $R$ of time $t$. The table below shows the rate as measured every 3 hours for a 24-hour period.

| $t$ (hours) | 0 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 |
|---|---|---|---|---|---|---|---|---|---|
| $R(t)$ (gal/hr) | 9.6 | 10.4 | 10.8 | 11.2 | 11.4 | 11.3 | 10.7 | 10.2 | 9.6 |

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_0^{24} R(t)\,dt$. Using correct units, explain the meaning of your answer in terms of water flow.

(b) Is there some time $t$, $0 < t < 24$, such that $R'(t) = 0$? Justify your answer.

(c) The rate of the water flow $R(t)$ can be approximated by $Q(t) = \frac{1}{79}(768 + 23t - t^2)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour time period. Indicate units of measure.

[1999 AB]

11. A particle moves along the $y$-axis with velocity given by $v(t) = t\sin(t^2)$ for $t \geq 0$.

    (a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

    (b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$?

(c) Given that $y(t)$ is the position of the particle at time $t$ and that $y(0) = 3$, find $y(2)$.

(d) Find the total distance traveled by the particle from $t = 0$ and $t = 2$.

[1999 AB]

12. Let $G(x) = \int_0^x \sqrt{16 - t^2}\, dt$.

    (a) Find $G(0)$.

    (b) Does $G(2) = G(-2)$? Does $G(2) = -G(-2)$?

(c) What is $G'(2)$?

(d) What are $G(4)$ and $G(-4)$?

13. The temperature on New Year's Day in Buffalo, New York, is given by

$$T(h) = -A - B\cos\left(\frac{\pi h}{12}\right),$$

where $T$ is the temperature in degrees Fahrenheit and $h$ is the number of hours from midnight $(0 \le h \le 24)$.

(a) The initial temperature at midnight was $-15^\circ$ F, and at Noon of New Year's Day it was $5^\circ$ F. Find $A$ and $B$.

(b) Find the average temperature for the first 10 hours.

(c) Use the trapezoid rule with 4 equal subdivisions to estimate $\int_6^{10} T(h)\, dh$. Using correct units, explain the meaning of your answer.

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(d) Find an expression for the rate that the temperature is changing with respect to $h$.

14. A particle moves along the $x$-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = 1 - \sin(2\pi t)$.

    (a) Find the acceleration $a(t)$ of the particle at any time $t$.

    (b) Find all values of $t$, $0 \leq t \leq 2$, for which the particle is at rest.

(c) Find the position $x(t)$ of the particle at any time $t$ if $x(0) = 0$.

15. Suppose that the function $f$ has a continuous second derivative for all $x$, and that $f(0) = 2$, $f'(0) = -3$, and $f''(0) = 0$. Let $g$ be a function whose derivative is given by $g'(x) = e^{-2x}(3f(x) + 2f'(x))$ for all $x$.

    (a) Write an equation for the line tangent to the graph of $f$ at the point where $x = 0$.

    (b) Is there sufficient information to determine whether or not the graph of $f$ has a point of inflection when $x = 0$? Explain your answer.

(c) Given that $g(0) = 4$, write an equation of the line tangent to the graph of $g$ at the point where $x = 0$.

(d) Show that $g''(x) = e^{-2x}(-6f(x) - f'(x) + 2f''(x))$. Does $g$ have a local maximum at $x = 0$? Justify your answer.

[1999 AB]