A.P. Calculus BC Test Three
Section One
Multiple-Choice
Calculators Allowed
Time—45 minutes

Number of Questions—15

The scoring for this section is determined by the formula

$$[C - (0.25 \times I)] \times 1.8$$

where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. The maximum possible points earned on this section is 27, which represents 50% of the total test score.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding choice on your answer sheet. Do not spend too much time on any one problem.

Good Luck!

NAME:
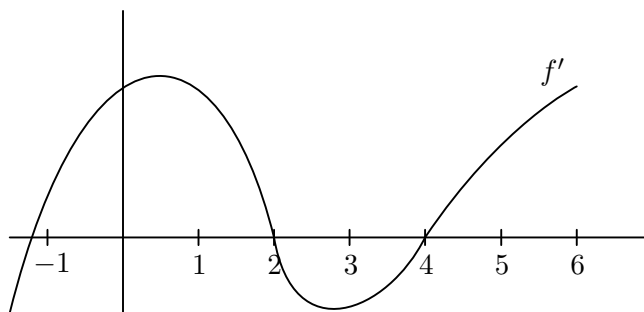
1. The graph of the first derivative f' for a function f is shown above. Which of the following statements *must* be true?

$$
\begin{array}{ll}
\text{I.} & f(4) = f(-1) \\
\text{II.} & f(2) > f(4) \\
\text{III.} & f''(2) < f''(4)
\end{array}
$$

A) I only

B) II only

C) II and III only

D) I and III only

E) I, II, and III

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2. Which of the following are true about the function f if its derivative is $f'(x) = (x-1)^2(4-x)$?

$$
\begin{array}{ll}
\text{I.} & f \text{ is decreasing for all } x < 4. \\
\text{II.} & f \text{ has a local maximum at } x = 1. \\
\text{III.} & f \text{ is concave up for all } 1 < x < 3.
\end{array}
$$

A) I only

B) II only

C) III only

D) II and III only

E) I, II, and III

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3. Consider $f(x) = 12 - x^2$ for $0 \le x \le 2\sqrt{3}$. Let $A(x)$ be the area of the triangle formed by the coordinate axes and the tangent to the graph of f at the point $(x, 12 - x^2)$. For what value of x is $A(x)$ a minimum?

A) 1.8

B) 1.9

C) 2.0

D) 2.1

E) 2.2

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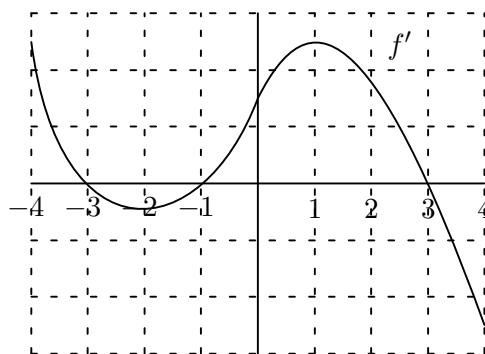
4. The graph of the derivative of a function f is shown above. Which of the following are true about the original function f ?

      I. f is increasing on the interval $(-2, 1)$.
     II. f is continuous at $x = 0$.
    III. f has an inflection point at $x = -2$.

    A) I only

    B) II only

    C) III only

    D) II and III only

    E) I, II, and III

5. The function $M(x) = x^4 - 4x^2$ has

    A) one relative minimum and two relative maxima

    B) one relative minimum and one relative maximum

    C) no relative minima and two relative maxima

    D) two relative minima and no relative maxima

    E) two relative minima and one relative maximum

6. Suppose a function f is differentiable for all x and $f(0) = 0$. If $g(x)$ is defined as $g(x) = f(x)\cos x$, which of the following statements must be true?

      I.   There exists a number c in the interval $\left(0, \frac{\pi}{2}\right)$ such that $g'(c) = 0$.
     II.   There exists a number c in the interval $\left(\frac{\pi}{2}, \pi\right)$ such that $g'(c) = 0$.
    III.   There exists a number c in the interval $\left(-\frac{\pi}{2}, 0\right)$ such that $g'(c) = 0$.

    A) I only

    B) II only

    C) I and II only

    D) I and III only

    E) I, II, and III

7. A function F is continuous on the interval $[-3, 3]$ and its first and second derivatives have the signs and zeros as given in the following table.

x	$(-3, -1)$	-1	$(-1, 0)$	0	$(0, 1)$	1	$(1, 3)$
$F'(x)$	$+$	0	$-$	$-$	$-$	0	$-$
$F''(x)$	$-$	$-$	$-$	0	$+$	0	$-$

Which of the following statements concerning F is true?

    A) F has a maximum at $x = 1$.

    B) F has a minimum at $x = 0$.

    C) F has an inflection point at $x = 1$.

    D) F has an inflection point at $x = -1$.

    E) F has no maxima or minima.

8. An object moves along the y-axis with coordinate position $y(t)$ and velocity $v(t) = \sqrt{t} - \cos(e^t)$ for $t \geq 0$. At time $t = 1$, the object is

    A) moving downward with negative acceleration.

    B) moving upward with negative acceleration.

    C) moving downward with positive acceleration.

    D) moving upward with positive acceleration.

    E) at rest.

9. What is the minimum velocity attained on the interval $0 \leq t \leq 3$ by a particle whose position is given by $s(t) = t^3 - 6t^2 + 9t - 1$?

    A) -3

    B) 0

    C) 3

    D) 6

    E) 9

10. The minimum distance from point $(5, 6)$ to the curve $y = x^2 + 1$ is

    A) 2.358

    B) 2.501

   C) 2.701

    D) 2.913

    E) 3.015

11. Which of the following functions fails to satisfy the conclusion of the Mean Value Theorem on the given interval?

    A) $3x^{2/3} - 1$ on $[-1, 2]$

    B) $|3x - 2|$ on $[1, 2]$

    C) $4x^3 - 2x + 3$ on $[0, 2]$

    D) $\sqrt{x - 2}$ on $[3, 6]$

    E) None of these

12. The function $F(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

    A) $F(0)$ does not exist.

    B) F is not continuous on $[-8, 8]$.

    C) $F(1)$ does not exist.

    D) F is not defined for $x < 0$.

    E) $F'(0)$ does not exist.

13. Find the sum of the values of a and b such that $f(x) = 2ax^2 + bx + 3$ has a relative extremum at the point $(-1, 2)$.

      A) $\frac{3}{2}$

      B) $\frac{5}{2}$

      C) 1

      D) -1

      E) None of these
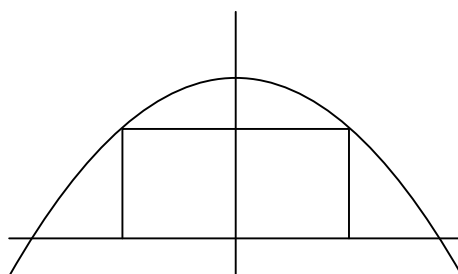
14. A rectangle with one side on the x-axis has its upper vertices on the graph of $y = \cos x$, as shown in the figure above. What is the maximum area of the rectangle?

      A) 0.561

      B) 0.617

      C) 0.860

      D) 1.122

      E) 1.234

15. The slope of the curve $y = x^2 - e^{-x}$ at its point of inflection is

      A) $-\ln 2$

      B) $-\ln 4$

      C) $2 - \ln 4$

      D) $2 + \ln 4$

      E) $e^2/2$