A.P. Calculus BC Test Five
Section One
Multiple-Choice
No Calculators

Time—35 minutes

Number of Questions—15

The scoring for this section is determined by the formula

$$[C - (0.25 \times I)] \times 1.8$$

where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. The maximum possible points earned on this section is 27, which represents 50% of the total test score.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding choice on your answer sheet. Do not spend too much time on any one problem.

Good Luck!

NAME:

1. If the substitution $u = \sqrt{x+1}$ is used, then $\displaystyle\int \frac{dx}{x\sqrt{x+1}} =$

A) $\displaystyle\int \frac{du}{u^2-1}$

B) $\displaystyle\int \frac{2\,du}{u^2-1}$

C) $\displaystyle 2\int \frac{du}{(u-1)(u+1)}$

D) $\displaystyle 2\int \frac{du}{u(u^2-1)}$

E) None of these

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2. $\displaystyle\int \frac{x}{\sqrt{9-x^2}}\,dx =$

A) $-\frac{1}{2}\ln\sqrt{9-x^2}+C$

B) $\arcsin(\frac{x}{3})+C$

C) $-\sqrt{9-x^2}+C$

D) $-\frac{1}{4}\sqrt{9-x^2}+C$

E) $2\sqrt{9-x^2}+C$

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3. $\displaystyle\int_0^1 xe^x\,dx =$

A) 1

B) -1

C) $2-e$

D) $\frac{1}{2}e^2-e$

E) $e-1$

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4. The average value of $y = \tan x$ in the interval $\left[0, \frac{\pi}{3}\right]$ is

A) $\ln 2$

B) $\frac{3}{\pi}(\ln 2)$

C) $-\ln 2$

D) $\frac{9}{\pi}$

E) $\frac{1}{2}\sqrt{3}$

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5. Evaluate $\lim_{x\to\infty} \frac{x^3}{e^x} =$

A) 3

B) 0

C) 6

D) 1

E) ∞

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6. If a function f is defined by $f(x) = \int_0^x \frac{1}{1+t^4}\, dt$, which of the following statements are true?

I. $f(x) = \frac{1}{2}$

II. The graph of f is concave down at $x = 3$

III. $f(x) + f(-x) = 0$ for all real numbers x

A) I only

B) II only

C) III only

D) II and III only

E) I, II, and III

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7. $\displaystyle\int_1^e \frac{\ln x}{x}\,dx =$

A) undefined
B) $\frac{1}{2}$
C) 2
D) $\frac{1}{2}(e-1)$
E) None of these

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8. $\displaystyle\int_e^\infty \frac{1}{x(\ln x)^2}\,dx =$

A) $\frac{1}{e}$
B) $-\frac{1}{e}$
C) e
D) 1
E) Divergent

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9. Consider the function g defined by $g(x) = \displaystyle\int_1^x (t^3 - 3t^2 + 2t)\,dt$. The number of relative extrema of g is

A) 1.
B) 2.
C) 3.
D) 4.
E) more than 4.

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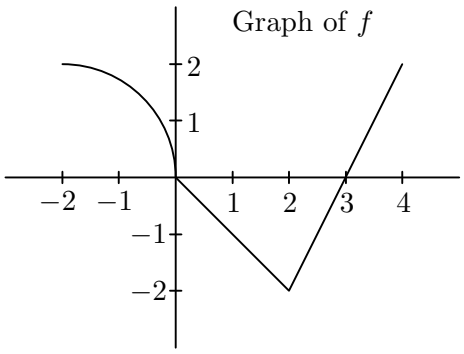

10. The graph of the function f shown above consists of a quarter circle and two line segments. Find $\int_{-2}^{4} f(x)\,dx$.

A) $-\pi - 2$

B) $2 - \pi$

C) $\pi - 2$

D) $\pi + 2$

E) $\pi + 4$

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11. What is the area of the region in the fourth quadrant between the x-axis, the y-axis, and the graph of $y = 3\ln x$?

A) $\frac{1}{3}$

B) 1

C) 3

D) 4

E) ∞

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12. A particle moves on the x-axis in such a way that its position at time t, for $t > 0$, is given by $x(t) = (\ln x)^2$. At what value of t does the velocity of the particle attain its maximum?

A) 1

B) $e^{1/2}$

C) e

D) $e^{3/2}$

E) e^2

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13. Given the differential equation

$$\frac{dP}{dt} = 20P(1600 - P)$$

with the initial condition $P(0) = 2000$, find $\lim_{t\to\infty} P(t)$.

A) 0
B) 800
C) 1600
D) 2000
E) ∞

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14. Let $p(x) = \dfrac{|x-4|}{x-4}$. Which of the following best describes the behavior of p at $x = 4$?

A) vertical asymptote
B) horizontal asymptote
C) cusp
D) jump discontinuity
E) oscillation

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15. $\lim_{x\to -1/2^-} \dfrac{2x^2 - 3x - 2}{2x + 1} =$

A) ∞
B) $-\infty$
C) 1
D) $\frac{3}{2}$
E) $-\frac{5}{2}$