A.P. Calculus BC Test Six
Section One
Multiple-Choice
No Calculators

Time—35 minutes

Number of Questions—15

The scoring for this section is determined by the formula

$$[C - (0.25 \times I)] \times 1.8$$

where C is the number of correct responses and I is the number of incorrect responses. An unanswered question earns zero points. The maximum possible points earned on this section is 27, which represents 50% of the total test score.

Directions: Solve each of the following problems, using the available space for scratch work. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding choice on your answer sheet. Do not spend too much time on any one problem.

Good Luck!

NAME:

1. Which of the following series is divergent?

A) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$

B) $\displaystyle\sum_{n=1}^{\infty} \frac{n+1}{n!}$

C) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$

D) $\displaystyle\sum_{n=1}^{\infty} \frac{\ln n}{2^n}$

E) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$

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2. The series $\displaystyle\sum_{n=1}^{\infty} u_n$ has terms for which $\lim_{n\to\infty} u_n = 0$. Which of the following statements is *always* true?

A) $\displaystyle\sum_{n=1}^{\infty} u_n$ converges to a finite sum.

B) $\displaystyle\sum_{n=1}^{\infty} u_n = 0.$

C) $\displaystyle\sum_{n=1}^{\infty} u_n$ does not diverge to infinty.

D) $\displaystyle\sum_{n=1}^{\infty} u_n$ is a positive series.

E) None of these

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3. The sequence $\left\{\sin \frac{n\pi}{6}\right\}$

A) is unbounded.

B) is monotonic.

C) converges to a number less than 1.

D) is bounded.

E) diverges to infinity.

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4. The Taylor series for $\ln(1+2x)$ about $x=0$ is

A) $2x - \dfrac{(2x)^2}{2} + \dfrac{(2x)^3}{3} - \dfrac{(2x)^4}{4} + \cdots$

B) $2x - 2x^2 + 8x^3 - 16x^4 + \cdots$

C) $2x - 4x^2 + 16x^3 + \cdots$

D) $2x - x^2 + \dfrac{8x^3}{3} - 4x^4 + \cdots$

E) $2x - \dfrac{(2x)^2}{2!} + \dfrac{(2x)^3}{3!} - \dfrac{(2x)^4}{4!} + \cdots$

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5. The slope of the spiral $r=\theta$ at $\theta=\pi/4$ is

A) $-\sqrt{2}$

B) -1

C) 1

D) $\dfrac{4+\pi}{4-\pi}$

E) undefined

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6. The radius of convergence of $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-2)^n}{n\cdot 3^n}$ is

A) 3

B) 2

C) 1

D) 0

E) ∞

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7. The area of the closed region bounded by the polar graph of $r = \sqrt{1 + \cos\theta}$ is given by

A) $\displaystyle\int_0^{2\pi} \sqrt{1 + \cos\theta}\ d\theta$

B) $\displaystyle\int_0^{\pi} \sqrt{1 + \cos\theta}\ d\theta$

C) $\displaystyle 2\int_0^{2\pi} (1 + \cos\theta)\ d\theta$

D) $\displaystyle\int_0^{\pi} (1 + \cos\theta)\ d\theta$

E) $\displaystyle 2\int_0^{\pi} \sqrt{1 + \cos\theta}\ d\theta$

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8. The radius of convergence of the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n(n+1)}{2^n(2n+1)}$ is

A) 4

B) 3

C) 2

D) 1

E) 0

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9. If a particle moves in the xy-plane so that at time $t > 0$ its position vector is $\left\langle \sin\left(3t - \frac{\pi}{2}\right),\ 3t^2 \right\rangle$, then at time $t = \pi/2$ the velocity vector is

A) $\langle -3, 3\pi \rangle$

B) $\langle -1, 3\pi \rangle$

C) $\langle -1, 2\pi \rangle$

D) $\langle 3, 2\pi \rangle$

E) $\langle 3, 3\pi \rangle$

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10. A particle moves in the xy-plane in such a way that its velocity vector is $\langle 1+t,\ t^3\rangle$. If the position vector at $t=0$ is $\langle 5,0\rangle$, then the position of the particle at $t=2$ is

A) $\langle 1,12\rangle$

B) $\langle 4,4\rangle$

C) $\langle 5,9\rangle$

D) $\langle 9,4\rangle$

E) $\langle 5,0\rangle$

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11. $\displaystyle\int \frac{8}{(x-1)(x+3)}\,dx =$

A) $\displaystyle 2\ln\left|\frac{x+3}{x-1}\right|+C$

B) $\displaystyle 2\ln\left|\frac{x-1}{x+3}\right|+C$

C) $\displaystyle 2\ln|(x+3)(x-1)|+C$

D) $\displaystyle 2\ln\left|\frac{1}{(x+3)(x-1)}\right|+C$

E) $\displaystyle 8\ln\left|\frac{1}{(x+3)(x-1)}\right|+C$

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12. If $f(x)=\dfrac{x-k}{x+k}$ and $k\neq 0$, then $f''(0)=$

A) $-4/k^2$

B) $-2/k$

C) 0

D) $2/k$

E) $4/k^2$

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13. What is the sum $\frac{5}{2}+\frac{5}{4}+\frac{5}{8}+\frac{5}{16}+\cdots$?

A) 2
B) $\frac{75}{16}$
C) $\frac{315}{64}$
D) 5
E) This series diverges

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14. Which of the following series is convergent?

I. $\sum_{n=1}^{\infty}\frac{1}{n^3}$ II. $\sum_{n=1}^{\infty}\frac{2}{n+1}$ III. $\sum_{n=1}^{\infty}\frac{3^n}{n\cdot 2^n}$

A) I only
B) II only
C) III only
D) I and III only
E) I, II, and III

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15. What is the third-degree Taylor polynomial for $f(x)=\tan x$ at $x=0$?

A) $x-\frac{x^3}{3!}$
B) $x+\frac{x^3}{3!}$
C) $x-\frac{x^3}{3}$
D) $x+\frac{x^3}{3}$
E) $x+\frac{2x^3}{3}$