DE Form 0287 - Local School District Course Submission Form

K-12 Mathematics Introduction

Georgia Mathematics focuses on actively engaging the student in the development of mathematical understanding by working independently and cooperatively to solve problems, estimating and computing efficiently, using appropriate tools, concrete models and a variety of representations, and conducting investigations and recording findings. There is a shift toward applying mathematical concepts and skills in the context of authentic problems and student understanding of concepts rather than merely following a sequence of procedures. In mathematics classrooms, students will learn to think critically in a mathematical way with an understanding that there are many different solution pathways and sometimes more than one right answer in applied mathematics. Mathematics is the economy of information. The central idea of all mathematics is to discover how knowing some things leads, via reasoning, to knowing more—without having to commit the information to memory as a separate fact. It is the reasoned, logical connections that make mathematics manageable. The implementation of the Georgia Standards of Excellence in Mathematics places the expected emphasis on sense-making, problem solving, reasoning, representation, modeling, representation, connections, and communication.

Advanced Finite Mathematics

Advanced Finite Mathematics will look at mathematics in four areas through the lens of both pure mathematics and applied mathematics: set theory, number theory, probability/combinatorics, and graph theory. There will be a strong focus on the presentation of mathematical ideas through both written and oral communication, particularly through logic and proofs. Mathematical proofs will be presented through an abstract approach that characterizes upper level mathematics courses. The goal is to give students the skills and techniques they will need as they study advanced mathematics at the college level. This is an alternative course for those students who do not wish to enroll in an Advanced Placement course, but who still wish to learn higher-level mathematics.

Mathematical Practices are listed with each grade's mathematical content standards to reflect the need to connect the mathematical practices to mathematical content in instruction.

The Standards for Mathematical Practice describe varieties of expertise that mathematics educators at all levels should seek to develop in their students. These practices rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these are the NCTM process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up*: adaptive reasoning, strategic competence, conceptual understanding (comprehension of mathematical concepts, operations and relations), procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately), and productive disposition (habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy).

1 Make sense of problems and persevere in solving them.

High school students start to examine problems by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems, and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. By high school, students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. They check their answers to problems using different methods and continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

2 Reason abstractly and quantitatively.

High school students seek to make sense of quantities and their relationships in problem situations. They abstract a given situation and represent it symbolically, manipulate the representing symbols, and pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Students use quantitative reasoning to create coherent representations of the problem at hand; consider the units involved; attend to the meaning of quantities, not just how to compute them; and know and flexibly use different properties of operations and objects.

3 Construct viable arguments and critique the reasoning of others.

High school students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases, and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. High school students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. High school students learn to determine domains to which an argument applies, listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments.

4 Model with mathematics.

High school students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. High school students making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

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5 Use appropriate tools strategically.

High school students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. High school students should be sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. They are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

6 Attend to precision. High school students try to communicate precisely to others by using clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

7 Look for and make use of structure. By high school, students look closely to discern a pattern or structure. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y. High school students use these patterns to create equivalent expressions, factor and solve equations, and compose functions, and transform figures.

8 Look for and express regularity in repeated reasoning.

High school students notice if calculations are repeated, and look both for general methods and for shortcuts. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, derive formulas or make generalizations, high school students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Standards for Mathematical Practice to the Standards for Mathematical Content

The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics should engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction.   The Standards for Mathematical Content are a balanced combination of procedure and understanding. Expectations that begin with the word "understand" are often especially good opportunities to connect the practices to the content. **Students who do not have an understanding of a topic may rely on procedures too heavily.** Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut. **In short, a lack of understanding effectively prevents a student from engaging in the mathematical practices.**

In this respect, those content standards that set an expectation of understanding are potential "points of intersection" between the Standards for Mathematical Content and the Standards for Mathematical Practice. These points of intersection are intended to be weighted toward central and generative concepts in the school mathematics curriculum that most merit the time, resources, innovative energies, and focus necessary to qualitatively improve the curriculum, instruction, assessment, professional development, and student achievement in mathematics.

Advanced Finite Mathematics | Content Standards

Logical Reasoning LR

Represent and interpret statements using logical symbolism.

MGSE.AFM.LR.1 Represent statements, including conditional, biconditional, and quantified statements, using truth tables to determine whether the statement is true or false.

MGSE.AFM.LR.2 Represent logic operations such as AND, OR, NOT, NOR, and XOR (exclusive OR) using logical symbolism, determine whether statements involving these operations are true or false, and interpret such symbols into English.

MGSE.AFM.LR.3 Determine whether a logical argument is valid or invalid, and determine whether a logical argument is a tautology or a contradiction.

MGSE.AFM.LR.4 Write and determine the truth of the negation, the converse, and the inverse of a conditional statement and understand that the contrapositive of a statement is logically equivalent to the statement.

Set Theory ST

Use set theoretic operations.

MGSE.AFM.ST.1 Calculate the union, intersection, difference, and Cartesian product of two sets. Determine whether two sets are equal. Determine whether one set is a subset of another. Determine the power set and the complement of a set.

MGSE.AFM.ST.2 Understand that a partitioning of a set is to split its elements into disjoint subsets.

MGSE.AFM.ST.3 Understand that a function on two sets is a bijective relation on two sets.

MGSE.AFM.ST.4 Given a relation on two sets, determine whether the relation is a function and determine the relation's inverse relation, if it exists.

MGSE.AFM.ST.5 Understand that an equivalence class is a partition of a set determined by an equivalence relation. Determine an equivalence class given the equivalence relation on a set.

MGSE.AFM.ST.6 Prove set relations, including DeMorgan's Laws and equivalence relations. For example, given two sets A and B, prove that the intersection of A and B is a subset of A and a subset of B; prove that the empty set is unique and is a subset of every set.

Use and interpret Boolean algebra.

MGSE.AFM.ST.7 Represent the dichotomy between "true" and "false" with 1s and 0s. Use 1s and 0s to calculate whether a statement is true or false by constructing Boolean logic circuits.

MGSE.AFM.ST.8 Convert binary and hexadecimal numbers into decimal, and convert from binary to hexadecimal, and vice versa. Use the method of 2's complement to subtract binary integers.

MGSE.AFM.ST.9 Prove statements in Boolean algebra.

Number Theory — NT

Use number-theoretic operations.

MGSE.AFM.NT.1 Apply the "divides" relation to positive integers. Calculate one integer modulo another integer; that is calculate a mod n for some integers a and n.

MGSE.AFM.NT.2 Determine the inverse of an integer for a certain modulus.

MGSE.AFM.NT.3 Calculate the floor and the ceiling of a real number.

Prove statements in number theory.

MGSE.AFM.NT.5 Prove statements involving properties of numbers. For example, prove that the sum of two rational numbers is rational; prove that if a is even and b is odd, then $(a^2 + b^2 + 1)/2$ is an integer; prove that any odd number squared is of the form $8k + 1$ for some integer k; prove that the square root of 2 is irrational.

MGSE.AFM.NT.6 Prove statements involving the floor and ceiling functions. For example, for every integer *n*, prove that the floor of *n*/2 is equal to *n*/2 if *n* is even and equal to (*n* – 1)/2 if *n* is odd.

MGSE.AFM.NT.7 Prove the Fundamental Theorem of Arithmetic, the Euclidean algorithm, and Fermat's Little Theorem.

Apply number theory.

MGSE.AFM.NT.8 Apply the RSA algorithm to encrypt a numerically coded message and to decode an encoded message.

Probability and Combinatorics PC

Calculate the probability of events.

MGSE.AFM.PC.1 Use the addition rule to count the number of outcomes in a disjoint set of sample spaces. Use the principle of inclusion-exclusion to count the number of outcomes in the union of space spaces.

MGSE.AFM.PC.2 Apply the axioms of probability to determine the probability of dependent and independent events, including use of the multiplication rule for independent events.

MGSE.AFM.PC.3 Apply probabilistic methods to determine the expected value of a random process.

MGSE.AFM.PC.4 Apply Bayes' Theorem to determine conditional probability.

Use methods of counting.

MGSE.AFM.PC.5 Calculate the number of permutations of a set with *n* elements. Calculate the number of permutations of *r* elements taken from a set of *n* elements.

MGSE.AFM.PC.6 Calculate the number of subsets of size *r* that can be chosen from a set of *n* elements. Recognize this number as the number of combinations "*n* choose *r*".

MGSE.AFM.PC.7 Calculate the number of combinations with repetitions of *r* elements from a set of *n* elements as $n + r – 1$ choose *r*.

Prove statements involving combinatorics.

MGSE.AFM.PC.8 Prove combinatorial identities. For example, prove that $n + 1$ choose *r* is equal to *n* choose $r – 1$ plus *n* choose *r*; prove that the sum of the entries in the *k*th row of Pascal's triangle is 2^k (where the first row is row 0).

MGSE.AFM.PC.9 Apply a combinatorial argument to prove the binomial theorem.

MGSE.AFM.PC.10 Use the pigeonhole principle to prove statements about counting.

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Graph Theory GT

Use and recognize graph properties.

MGSE.AFM.GT.1 Identify simple graphs, complete graphs, complete bipartite graphs, and trees. Identify graphs that have Euler and Hamiltonian cycles.

MGSE.AFM.GT.2 Determine the complement of a graph and the line graph of a graph.

MGSE.AFM.GT.3 Use the adjacency matrix of a graph to determine the number of walks of length n in a graph.

Prove statements in graph theory.

MGSE.AFM.GT.4 Prove statements about graph properties. For example, prove that a graph has an even number of vertices of odd degree; prove that a graph has an Euler cycle if and only if the graph is connected and every vertex has even degree; prove that any tree with n vertices has $n – 1$ edges.

Apply graph theory.

MGSE.AFM.GT5 Prove that every connected graph has a minimal spanning tree.

MGSE.AFM.GT6 Use Kruskal's algorithm and Prim's algorithm to determine the minimal spanning tree of a weighted graph.

Methods of Proof MP

MGSE.AFM.MP.1 Use a counterexample to disprove a statement. For example, the statement "there exists a positive integer n such that $n^2 + 3n + 2$ is prime" is disproved by the counterexample $n = 3$.

MGSE.AFM.MP.2 Prove statements directly from definitions and previously proved statements. For example, using the definition of "rational number", prove that every integer is rational; using the definition of "divides", prove that if a divides b and b divides a, then a equals b.

MGSE.AFM.MP.3 Prove statements indirectly by proving the contrapositive of the statement. For example, prove that if the square of an integer is even, then the integer is even, by instead proving that if an integer is odd, then its square is odd.

MGSE.AFM.MP.4 Apply the method of *reductio ad absurdum* ("proof by contradiction") to prove statements, such as proving that there are infinitely many prime numbers.

MGSE.AFM.MP.5 Use the method of mathematical induction to prove statements involving the positive integers. For example, prove that 3 divides $2^{2n} – 1$ for all positive integers n; prove that $1 + 2 + 3 + … + n = n(n + 1)/2$; prove that a given recursive sequence has a closed form expression.