Discrete Mathematics Test One

Number of Questions—16

Total points—146

Point-values for each problem are in brackets in the right-hand margin. You may answer as many of the 16 problems as you wish.

Directions: Solve each of the following problems on this test, using the available space to show your work.

Good Luck!

NAME:

1. Use DeMorgan's Laws to write the negation of "Sam is an orange belt and Katie is a red belt." [4]

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2. Write the converse and the contrapositive of "If it is sunny, then I will go swimming." [6]

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3. Write the truth table for $\sim(r \Rightarrow \sim q) \vee (p \wedge \sim r)$. [10]

Continued on the next page.

4. Consider the truth table below. [20]

p	q	??
T	T	F
T	F	F
F	T	T
F	F	F

(a) Find a statement with this truth table.

(b) Find a digital logic curcuit whose input/output table corresponds with this truth table.

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5. Consider the following statement: "If the solution is boiling, then its temperature must be at least 150°C." Assuming that this statement is true, circle any of the following that must also be true. [15]

A) If the temperature of the solution is at least 150°C, then the solution is boiling.

B) If the temperature of the solution is less than 150°C, then the solution is not boiling.

C) The solution will boil only if its temperature is at least 150°C.

D) If the solution is not boiling, then its temperature is less than 150°C.

E) A necessary condition for the solution to boil is that its temperature be at least 150°C.

F) A sufficient condition for the solution to boil is that its temperature be at least 150°C.

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6. A set of premises is given below. What is the logical conclusion? [10]

$$p \vee q$$
$$q \Rightarrow r$$
$$p \wedge s \Rightarrow t$$
$$\sim r$$
$$\sim q \Rightarrow u \wedge s$$

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7. Use 8-bit binary representations to compute $89 + (-55)$. [8]

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8. Convert $39EB_{16}$ to decimal. [4]

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9. Consider the following statement: "$\exists\, x \in \mathbb{R} \ni x^2 = 2$." Circle any of the following that are equivalent to this statement. [15]

A) The square of each real number is 2.

B) Some real numbers have square 2.

C) The number x has square 2, for some real number x.

D) If x is a real number, then $x^2 = 2$.

E) Some real number has square 2.

F) There is at least one real number whose square is 2.

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10. Write the negation of "$\forall\ x \in \mathbb{R}, x^2 \geq 1 \Rightarrow x > 0$." [5]

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11. Consider each of the following arguments. If the following argument is valid, state whether it is valid by modus ponens or modus tonens; if the argument is invalid, state whether the error exhibited is the converse or inverse error. [10]

All honest people pay their taxes.
Jonathan is not honest.
Therefore, Jonathan does not pay his taxes.

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12. Using the symbols $\forall$ and $\exists$, rewrite the statement "Everybody is older than somebody." [8]

Continued on the next page.

13. Prove or provide a counterexample: For any positive integer n, n^2 is of the form $4k$ or $4k+1$ for some integer k. [8]

14. Let $a, b, c \in \mathbb{Z}$. Prove: If $a \nmid bc$, then $a \nmid b$. [8]

15. Prove: $\forall\, n \in \mathbb{Z}, \left\lfloor \frac{n}{2} \right\rfloor = \begin{cases} \frac{n}{2} & n \text{ even} \\ \frac{n-1}{2} & n \text{ odd} \end{cases}$ [8]

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16. If $12a = 25b$, does $12|b$? Does $25|a$? Explain. [7]