

Discrete Mathematics Test One

Number of Questions—15

Total points—113

Point-values for each question are in brackets next to the problem number. You may answer as many of the 15 problems as you wish.

Directions: Solve each of the following problems on this test, using the available space to show your work.

Good Luck!

NAME:

[5] 1. Compute $153 + 154 + 155 + \cdots + 472$.

[6] 2. Consider the equation $x_1 + x_2 + x_3 + x_4 = 30$. Out of all possible nonnegative solutions, what is the probability of obtaining a solution where each x_i is greater than 2?

[10] 3. Prove: $5 \mid (7^n - 2^n)$ for all $n \in \mathbb{Z}^+$.

[10] 4. Prove: $1 + 3 + 5 + \cdots + (2n - 1) = n^2$ for all $n \in \mathbb{Z}^+$.

- [10] 5. Suppose the only currency in use were \$3 bills and \$8 bills. Prove that every dollar amount greater than \$13 could be made from a combination of these bills.

[5] 6. Assuming Row 0 is the initial 1, find the sum of the entries in the 19th row of Pascal's Triangle.

[5] 7. Suppose A and B are events in a sample space S and suppose that $P(A) = 0.6$, $P(\overline{B}) = 0.4$, and $P(A \cap B) = 0.2$. What is $P(A \cup B)$?

[6] 8. Consider the following statements. Each statement is either true or false; circle the statements that are *true*.

A) $\mathbb{Z} \subseteq \mathbb{Q}$

B) $\mathbb{Z}^- \cap \mathbb{Z}^+ = \emptyset$

C) $\mathbb{Z} - \mathbb{Z}^+ = \mathbb{Z}^-$

D) $\overline{\mathbb{Z}} = \mathbb{Q}$

E) $\mathbb{Z} \cup \mathbb{R} = \mathbb{R}$

F) $\mathbb{Q} \cap \mathbb{Z} = \mathbb{Z}$

[8] 9. Let A and B be sets. Prove $B - A = B \cap \overline{A}$.

[8] 10. Let A and B be sets. Prove $(A - B) \cap B = \emptyset$.

- [8] 11. A combination lock requires three selections of numbers, each from 1 through 30.
- (a) How many different combinations are possible?
 - (b) Suppose no number may be used twice. How many different combinations are possible?

[10] 12. Let $n, r \in \mathbb{Z}^+$ with $r < n$. Prove:
$$\binom{n}{r+1} = \frac{n-r}{r+1} \binom{n}{r}.$$

- [8] **13.** (a) How many ways can the letters of the word QUIET be arranged?
- (b) How many ways can the letters of the word QUIET be arranged if the Q and the U must remain next to each other in the order QU ?

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- [6] **14.** From a standard 52-card deck, what is the probability of getting a poker hand consisting of four-of-a-kind?

- [8] 15. (a) How many distinguishable ways can the letters of the word REPREHENSIBLE be arranged?
- (b) How many distinguishable arrangements of the letters of the word REPREHENSIBLE begin with S and end with E ?

Question	Points	Score
1	5	
2	6	
3	10	
4	10	
5	10	
6	5	
7	5	
8	6	
9	8	
10	8	
11	8	
12	10	
13	8	
14	6	
15	8	
Total:	113	