Discrete Mathematics Test Three

Number of Questions—14

Total points—124

Point-values for each question are in brackets next to the problem number. You may answer as many of the 14 problems as you wish.

Directions: Solve each of the following problems on this test, using the available space to show your work.

Good Luck!

NAME:

[6] **1.** Let $A = \{1,2,3,4,5\}$, let $X \in \mathscr{P}(A)$, and let function $F : \mathscr{P}(A) \to \mathbb{Z}$ be defined as

$$F(X) = \begin{cases} 0 & \text{if } N(X) \text{ is even} \\ N(X) & \text{if } N(X) \text{ is odd} \end{cases}$$

What is the range of F?

[3] **2. (a)** In a group of 30 people, at least how many must have been born in the same month?

[6] **(b)** Suppose X and Y are finite sets, X has more elements than Y, and $F : X \to Y$ is a function. Must this function be a bijection? Explain.

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[8] **3.** Let $A = \{3, 4, 5\}$ and $B = \{4, 5, 6\}$. Define the relation R as the following:

$$\forall (x, y) \in A \times B, \ \ x \, R \, y \iff x < y.$$

State explicitly which ordered pairs are in R and R^{-1}.

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[10] **4.** Prove: If $f : X \to Y$ and $g : Y \to Z$ are both one-to-one functions, then $g \circ f$ is one-to-one.

[12] **5.** Consider the relation R defined as follows: $\forall\, x, y \in \mathbb{R},\ \ x\ R\ y \iff x - y$ is an integer. Prove that R is an equivalence relation.

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6. Let S be the set of all strings consisting of As and Bs and let $s \in S$.

[6] **(a)** Define $N : S \to \mathbb{Z}$ by

$$N(s) = \text{the number of As in } s.$$

Is N a bijection? Prove or give a counterexample.

[6] **(b)** Define $L : S \to \mathbb{Z}$ by

$$L(s) = \text{the number of characters in } s.$$

Define $T : \mathbb{Z} \to \{0, 1, 2\}$ by

$$T(n) = n \mod 3.$$

Does $T \circ L$ have an inverse? Explain.

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[6] **7.** Determine $679^{88} \mod 89$.

[6] **8.** Use Wilson's Theorem to prove that for any prime p, $(p-2)! \equiv 1 \mod p$.

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[10] **9.** Use the Euclidean Algorithm to find the greatest common divisor of 5328 and 1295, then express that greatest common divisor as a linear combination of 5328 and 1295.

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[5] **10.** **(a)** Find an inverse for $41 \mod 660$.

[5] **(b)** Find the least positive solution of $41x \equiv 125 \mod 660$.

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[9] **11.** Let the alphabet be encoded in two-digit blocks: $A = 01, B = 02, \ldots, Z = 26$. With the public key 91 and the value $e = 5$, use an RSA cipher to encrypt the message HELLO.

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12. Draw the graph K_6.

[4] **(a)** Does K_6 have a Euler cycle?

[4] **(b)** Does K_6 have a Hamiltonian cycle?

13. For the following, draw a graph with the given properties or prove that no such graph exists.

[4] **(a)** four vertices of degrees 1, 1, 3, 4

[4] **(b)** four vertices of degrees 1, 1, 3, 3

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[10] **14.** Prove that K_n has $\frac{1}{2}n(n-1)$ edges for any $n \geq 1$.

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Question	Points	Score
1	6	
2	9	
3	8	
4	10	
5	12	
6	12	
7	6	
8	6	
9	10	
10	10	
11	9	
12	8	
13	8	
14	10	
Total:	124	