The First Computations

The History of Mathematics, Part 2

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Egyptian Arithmetic
Babylonian Numerals and Arithmetic
The Method of False Position
Chinese Numerals
Greek Numerals
Homework

Outline

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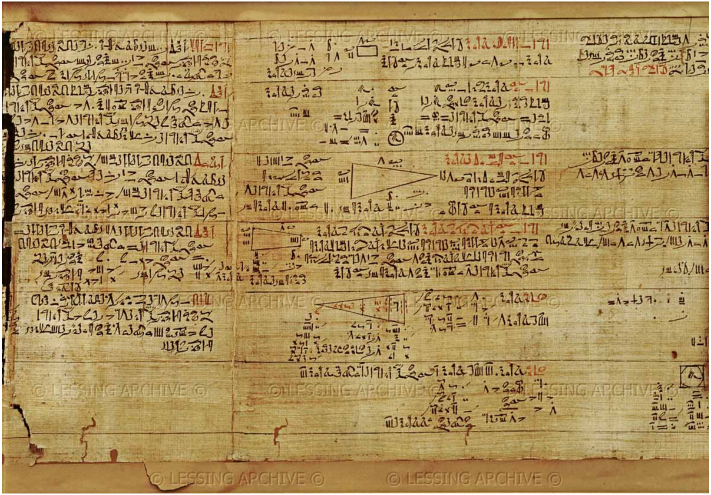
- Info from Moscow papyrus (c.1850 BC) and Rhind papyrus (c.1650 BC)
- Hieroglyphic addition and subtraction similar to present
- Hieratic arithmetic may have relied on tables
- Multiplication and division achieved by *doubling*

The Rhind Papyrus



Doubling

Multiply 12 by 25.

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1	12

Doubling

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1	12
2	24
4	48
8	96
16	192

Doubling

Multiply 12 by 25.

$1'$	12
2	24
4	48
$8'$	96
$16'$	192
25	300

Doubling

Divide 858 by 26.

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Divide 858 by 26.

1	26

Doubling

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1	26
2	52
4	104
8	208
16	416
32	832

Doubling

Divide 858 by 26.

1	$26'$
2	52
4	104
8	208
16	416
32	$832'$
33	858

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Egyptian Fractions

- Only fractions were unit fractions – fractions of the form $1/n$.
- Notation: dot or accent or bar over the number
- Example: $\dot{5} = 1/5$
- Special symbol for $2/3$; only non-unit fraction
- Extensive tables; notably $2/n$ fractions

Fractions and Doubling

Complete $2/3 + 1/15$ to 1.

Fractions and Doubling

Complete $2/3 + 1/15$ to 1.

1	15
$1/3$	5
$1/5$	3
$1/15$	1

Fractions and Doubling

Complete $2/3 + 1/15$ to 1.

$$\begin{array}{rl} 1 & 15 \\ 1/3 & 5 \\ 1/5 & 3' \\ 1/15 & 1' \\ \hline 1/5 + 1/15 & 4 \end{array}$$

Fractions and Doubling

Multiply by 7 to get 25.

Fractions and Doubling

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Multiply by 7 to get 25.

$$
\begin{array}{cccc}
 & & 1 & 7 \\
 & & 1/2 & 3 + 1/2' \\
1 & 7 & 1/7 & 1 \\
2 & 14 & 1/14 & 1/2' \\
\hline
3 & 21 & 1/2 + 1/14 & 4
\end{array}
$$

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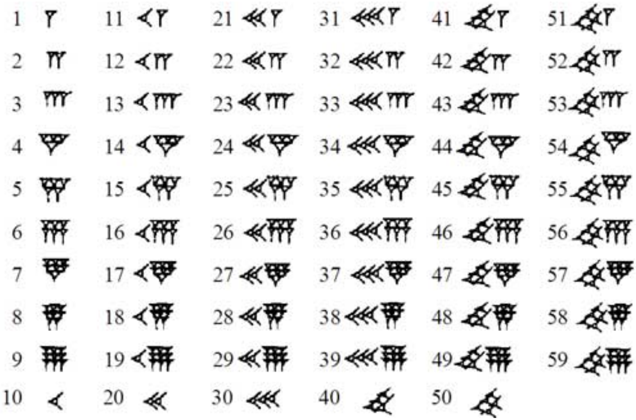
Babylonian Numerals

- Used a base-60 positional system
- Cuneiform writing
- Only two numerals

Babylonian Numerals



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Babylonian Arithmetic

- Info found on clay tablets c.2100-1600 BC
- Addition and subtraction similar to present procedure
- Used tables to multiply and divide
- Extensive tables of reciprocals since division by n was multiplication by $1/n$
- This may explain why the base was 60

Babylonian Table of Reciprocals

Babylonian tablet (BM 106444)

Babylonian Table of Reciprocals

Translation

2	30	16	3,45	45	1,20
3	20	18	3,20	48	1,15
4	15	20	3	50	1,12
5	12	24	2,30	54	1,6,40
6	10	25	2,24	1	1
8	7,30	27	2,13,20	1,4	56,15
9	6,40	30	2	1,12	50
10	6	32	1,52,30	1,15	48
12	5	36	1,40	1,20	45
15	4	40	1,30	1,21	44,26,40

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False Position

Given a problem such as

> Find a number so that the sum of itself and its quarter become 15.

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Given a problem such as

Find a number so that the sum of itself and its quarter become 15.

- Guess a solution; say 4
- Compute the problem assuming 4 is the solution:

$$4 + \frac{1}{4} \cdot 4 = 5$$

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- But the result should be 15
- Note that $5 \times 3 = 15$

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- Compute the problem assuming 4 is the solution:

$$4 + \frac{1}{4} \cdot 4 = 5$$

- But the result should be 15
- Note that $5 \times 3 = 15$
- Therefore, multiply the guess by 3
- Answer is 12

False Position

Why does this work?
Given a problem $p(x) = n$, where $p(x)$ is linear, we

- Guess a solution, say a
- Compute $p(a)$
- Then since $\frac{x}{a} = \frac{p(x)}{p(a)}$ and $p(x) = n$,

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Given a problem $p(x) = n$, where $p(x)$ is linear, we

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- Then since $\dfrac{x}{a} = \dfrac{p(x)}{p(a)}$ and $p(x) = n$,
- Solution is $a \times \dfrac{p(x)}{p(a)} = a \times \dfrac{n}{p(a)}$

False Position

Translated from an ancient Babylonian tablet:

> A number and its one-seventh. This is added to one-eleventh of itself. Result 60. Find the number.

False Position

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Mathematical translation:

$$\text{Solve } x + \frac{1}{7}x + \frac{1}{11}\left(x + \frac{1}{7}x\right) = 60.$$

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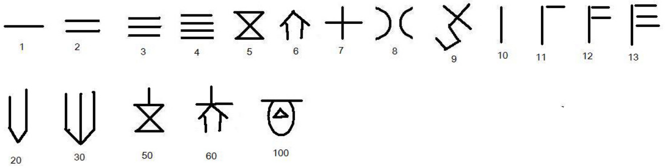
Chinese Numerals

- Two kinds:
  - *oracular*, additive notation
  - *rods*, positional base-10 notation

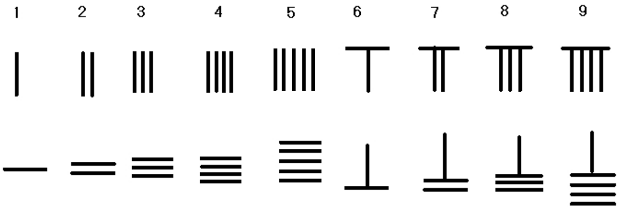
Oracular Chinese Numerals

Earliest known use from 1045 BC



Rod Chinese Numerals

Earliest known use from 4th century BC



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Greek Numerals

- Two kinds: *acrophonic* and *alphabetic*, both additive
- Alphabetic numerals are letters
- Distinguish numbers from words through context

Greek Acrophonic Numerals

Used as far back as 1000 BC

Greek Acrophonic Numerals and Example

Greek Alphabetic Numerals

Used from 4th century BC

α	β	γ	δ	ε	Ϝ	ζ	η	θ
1	2	3	4	5	6	7	8	9
ι	κ	λ	μ	ν	ξ	ο	π	Ϙ
10	20	30	40	50	60	70	80	90
ρ	σ	τ	υ	ϕ	χ	ψ	ω	ϡ
100	200	300	400	500	600	700	800	900

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- Last-Minute Problems, #1 – due February 1
- Using false position for systems of equations; *Math Through the Ages*, Sketch 9

Next: Two Mysteries