Two Mysteries: Pyramids and Greeks

The History of Mathematics, Part 3

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Outline

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Geometry of the Everyday

- Distance
- Simple shapes
- Vertical, parallel, perpendicular
- Discs of the sun and moon
- Spirals of coiled rope
- Cylindrical tree trunks
- Surfaces and solids of revolution
- Symmetry
- Etc…

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Called *subconscious geometry*

Inductive Geometry

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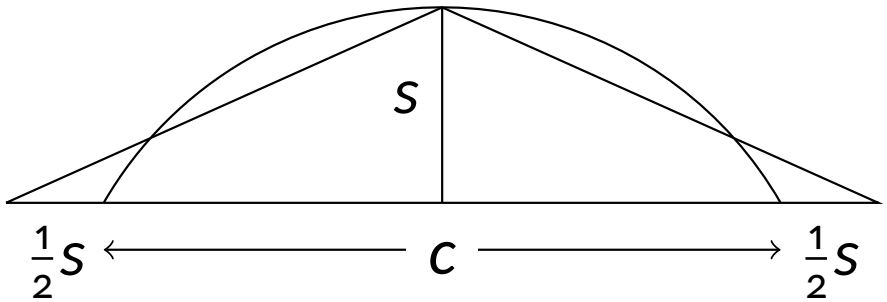
extract a general principle.

- Arose in “pockets” of civilization:
  - China, 5000 to 3000 BC
  - Babylon
  - Greece
- Before 600 BC, all recorded geometry is inductive

Inductive Geometry Example

This example is from the Chinese *Arithmetic in Nine Sections*, 213 BC (copy of a 1000 BC work)



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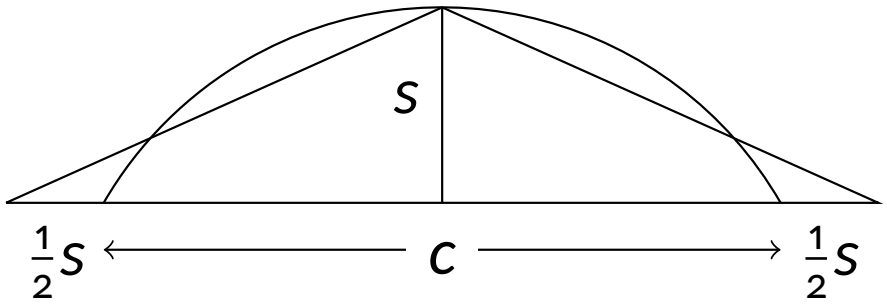

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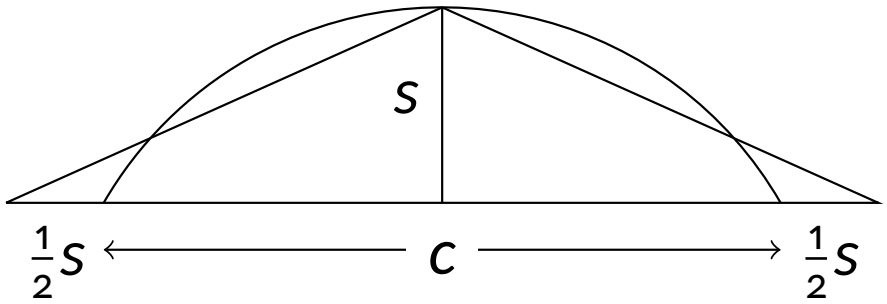

- c is the length of a chord of a circle
- s is the length of the sagitta
- Extend c on each side equal to $\frac{1}{2}s$
- Circular segment is approximately equal to isosceles triangle
- Thus, $A = \frac{1}{2}s(c + s)$

Implications

What if the chord c is a diameter?

- Then $s = r$ and $c = 2r$
- Chinese formula gives

$$A = \frac{1}{2}s(c+s) = \frac{1}{2}r(2r+r) = \frac{1}{2}(3r^2)$$

- Actual semicircular area is $\frac{1}{2}(\pi r^2)$

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Implication: $\pi \approx 3$

Approximations to π

- Chinese used $\pi = 3$
- Babylonians used $\pi = 3$ although one tablet c.1500 BC has $\pi = 3\frac{1}{8}$
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- Persisted for centuries

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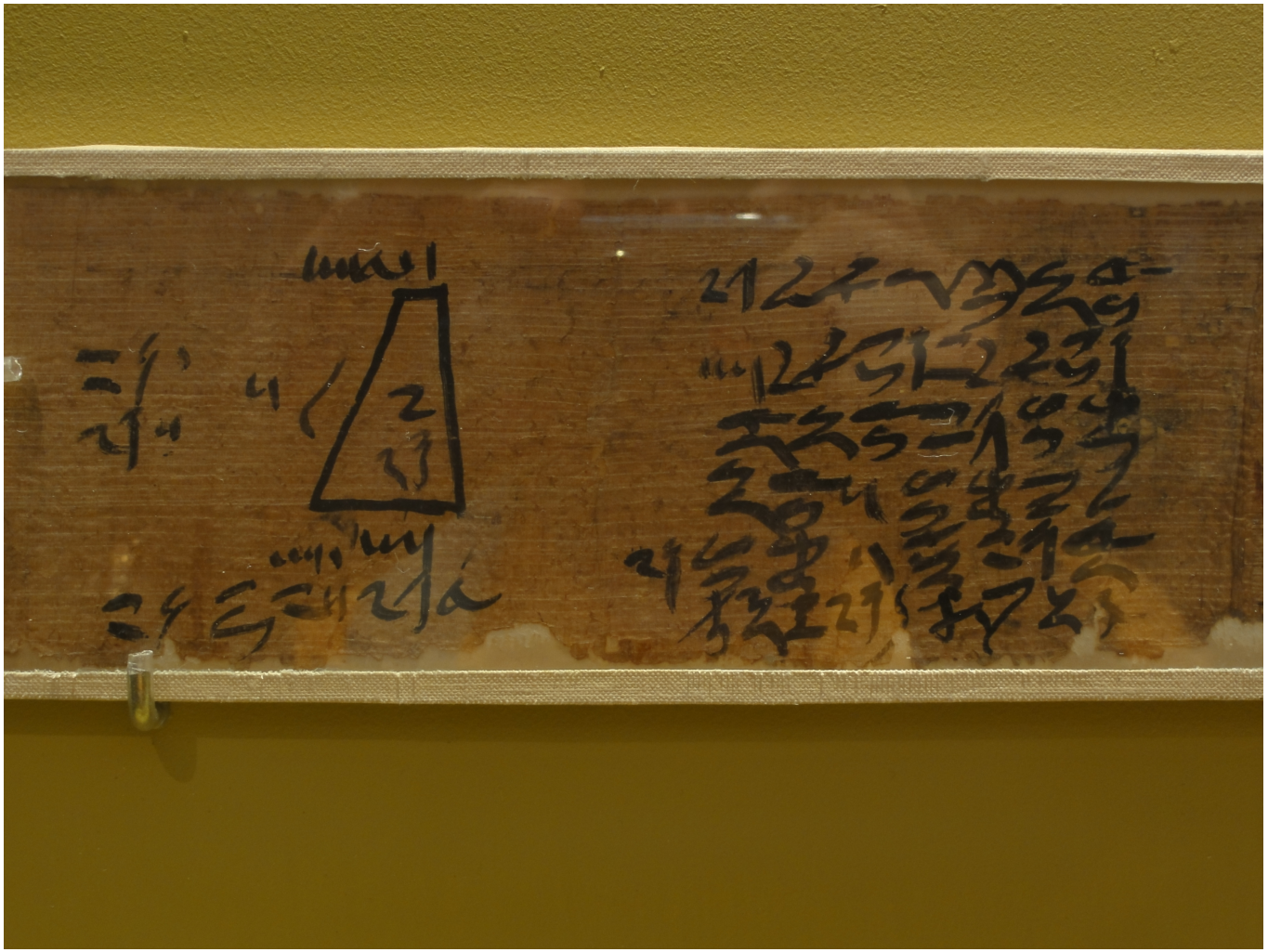
- Egyptians used $\pi = 3$ but knew that was not accurate
- Persisted for centuries
- 1 Kings 7:23 has $\pi = 3$:

"And he made a molten sea, ten cubits from the one brim to the other: it was round all about, and his height was five cubits: and a line of thirty cubits did compass it round about."

Outline

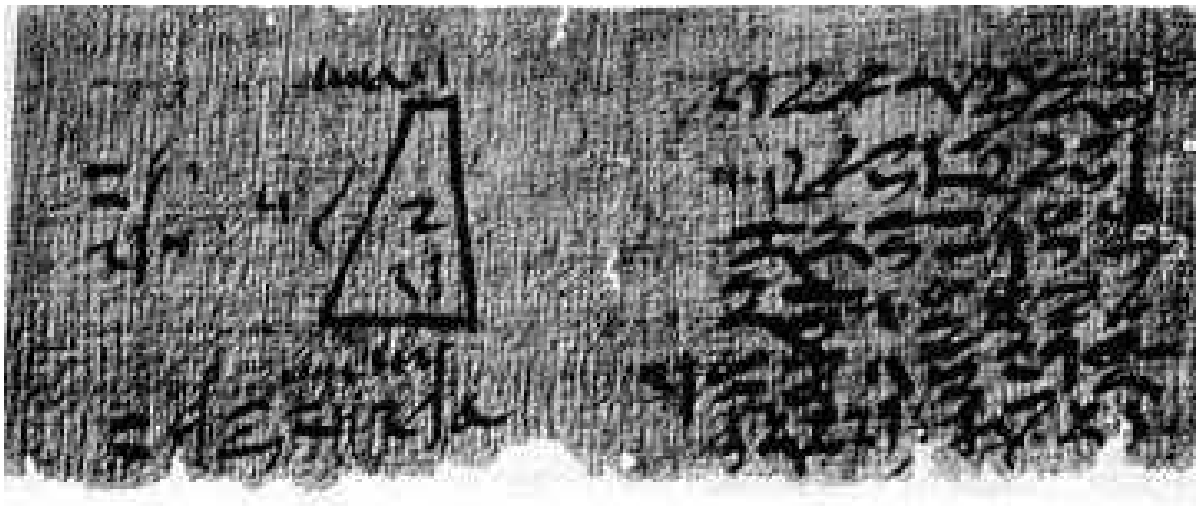
Moscow Papyrus, Problem 14

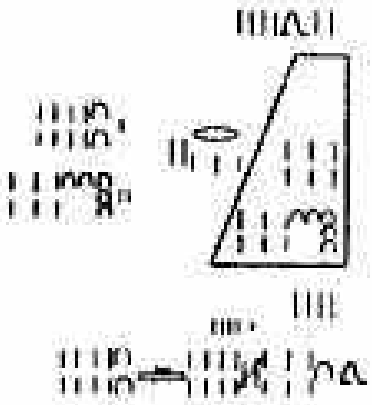


Moscow Papyrus, Problem 14

"You are given a truncated pyramid of 6 for the vertical height by 4 on the base by 2 on the top. You are to square this 4, result 16. You are to double 4, result 8. You are to square 2, result 4. You are to add the 16, the 8, the 4, result 28. You are to take one-third of 6, result 2. You are to take 28 twice, result 56. See, it is 56. You will find it right."

Moscow Papyrus, Problem 14





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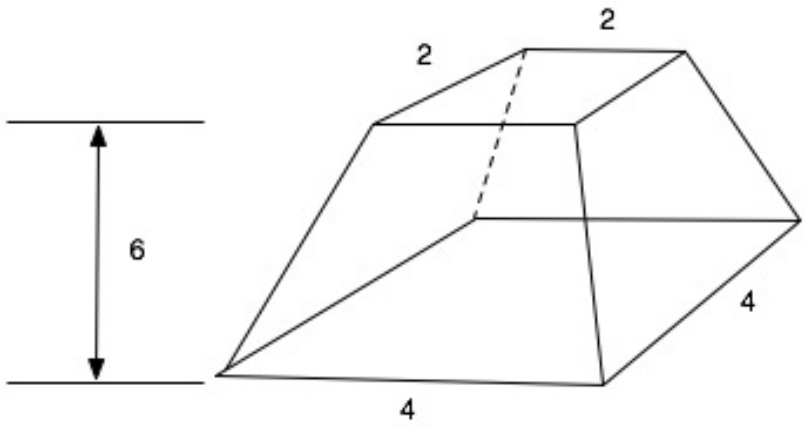

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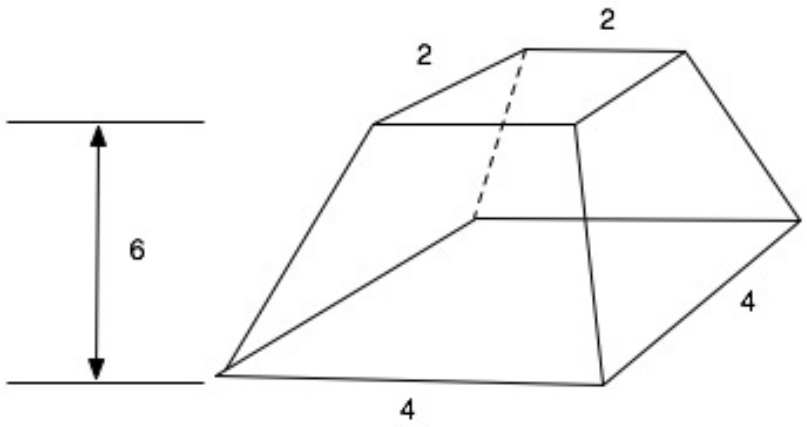

Let $b = 2$, $a = 4$, and $h = 6$.

You are to square this 4, result 16.	a^2
You are to double 4, result 8.	ab
You are to square 2, result 4.	b^2
Add the 16, the 8, the 4, result 28.	$a^2 + ab + b^2$
You are to take one-third of 6, result 2.	$\frac{1}{3}h$
You are to take 28 twice, result 56.	$\frac{1}{3}h(a^2 + ab + b^2)$

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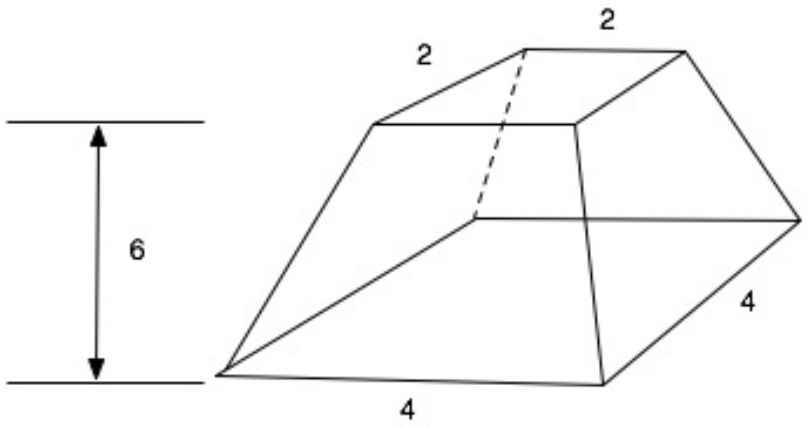

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You are to take 28 twice, result 56.	$\frac{1}{3}h(a^2 + ab + b^2)$

Thus, $V = \frac{1}{3}h\left(a^2 + ab + b^2\right)$.

This is the Greatest Pyramid

- Egyptians had $\frac{1}{3}h(a^2 + ab + b^2)$
- Babylonians had $\frac{1}{2}h(a^2 + b^2)$ (incorrect)
- Chinese had $\frac{1}{6}h\left[(2a + b)a + (2b + a)b\right]$

What if $b = 0$?

Outline

The Modern Mediterranean

Greece c.500 BC

The First Mathematician

Thales of Miletus

624 BC-547 BC

"Nothing is more active than thought, for it travels over the universe, and nothing is stronger than necessity for all must submit to it."

Thales

- Spent time as a merchant
- Became wealthy enough for travel and study
- Studied Egyptian and Babylonian mathematics
- Reputation as a statesman, counselor, engineer, businessman, philosopher, mathematician, astronomer
- Predicted solar eclipse of 585 BC
- “Everything is water”: first person to give non-mythological reasons for physical phenomena
- Originator of *deductive geometry*

Thales' Mathematics

Thales is credited with proving following.

1. A circle is bisected by any diameter.
2. The base angles of an isosceles triangle are equal.
3. Vertical angles formed by two intersecting lines are equal.
4. Two triangles are congruent of they have two angles and the included side in each respectively equal.
5. An angle inscribed in a semicircle is a right angle.

Impact

- Thales' arguments were the first example of mathematical reasoning removed from the computational
- Development of logic via geometry
- Geometry assumed first position in all philosophical education
- Thales is considered the father of natural philosophy

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Why the Greeks?

Four possible explanations

- Preference for deductive reasoning

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Four possible explanations

- Preference for deductive reasoning
- Love of beauty
- Class structure of society
- Consequence of economic and political change

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Homework

- The Beginnings of Mathematics;
  Math Through the Ages, pages 1-14.

Next: The First Great Theorem