

- (a) Find the parametric equation of the line that passes through  $(1, 2, -3)$  and  $(-5, 0, 1)$ .  
(b) Find the parametric equation of the line that passes through  $(1, 2, 4)$  and is parallel to  $x = 1 - 3t$ ,  $y = 1 + 2t$ ,  $z = 7t$ .  
(c) Find the equation of the plane through  $(1, 2, 4)$  and normal to  $x = 1 - 3t$ ,  $y = 1 + 2t$ ,  $z = 7t$ .  
(d) Find the distance from the point  $(-3, 0, 1)$  to the plane  $2x + 3y - z = 0$ .

- Let  $A$  be an  $n$ -square matrix with eigenvalues  $\lambda_1, \dots, \lambda_n$ .

(a) Prove that

$$\prod_{i=1}^n \lambda_i = \det A;$$

that is, prove that the product of the eigenvalues of  $A$  is equal to the determinant of  $A$ .

(b) Prove that

$$\sum_{i=1}^n \lambda_i = \sum_{i=1}^n a_{ii};$$

that is, prove that the sum of the eigenvalues of  $A$  is equal to the sum of the entries on the main diagonal of  $A$ .

- Determine whether the matrix  $M = \begin{bmatrix} 2/7 & 3/7 & 6/7 \\ 6/7 & 2/7 & -3/7 \\ 3/7 & -6/7 & 2/7 \end{bmatrix}$  is orthogonal, then find its eigenvalues.

- Let  $B$  and  $C$  be two  $n$ -square matrices and let  $A$  be an orthogonal matrix such that  $B = A^{-1}CA$ . Then  $B$  is said to be *orthogonally congruent* to  $C$ . Prove that if  $B$  is orthogonally congruent to  $C$ , then  $C$  is orthogonally congruent to  $B$ .

- Find the standard form for each of the following conic sections.

(a)  $5x^2 + 4xy + 3y^2$

(b)  $7x^2 + 2xy - y^2$

- Prove the Cauchy-Schwartz Inequality.

- Prove the Triangle Inequality.

- Let  $T$  map  $V^3$  into  $V^3$  and have the matrix  $Q = \begin{bmatrix} 3 & 1 & 2 \\ 1 & 0 & 1 \\ 5 & 2 & 3 \end{bmatrix}$ .

(a) Find the kernel of  $T$ , and determine whether  $T$  is one-to-one.

(b) Find the range of  $T$ , and determine whether  $T$  is onto.

- Let  $\mathbf{u} = \langle 2, 0, 3, 4, 6 \rangle$ ,  $\mathbf{v} = \langle 6, 3, 2, 1, 0 \rangle$ , and  $\mathbf{w} = \langle 3, 3, 3, 3, 3 \rangle$  be vectors in  $\mathbb{E}^5$ .

(a) Find the lengths of the sides of the triangle formed by  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{w}$ .

(b) Find cosines of the angles in the triangle formed by  $\mathbf{u}$ ,  $\mathbf{v}$  and  $\mathbf{w}$ .

- Prove that if  $A$  is an  $n$ -square orthogonal matrix, then each row vector of  $A$  is a unit vector.