1. Evaluate
$$\iint_R \frac{\sin y}{y}\, dy\, dx$$
where R is the triangle with vertices at the origin, $(0,\pi)$, and (π,π).

2. Using the substitution $u = 1+x+y$ and $v = x-y$, evaluate
$$\iint_R \frac{(x-y)^2}{1+x+y}\, dx\, dy$$
where R is the trapezoid in the first quadrant bounded by the lines $x+y=1$ and $x+y=2$.

3. Find the volume under the graph of $f(x,y) = x^2+y^2$ and above the circle $x^2+y^2 \le 1$.

4. Evaluate $\int_{(0,0)}^{(2,4)} y\ dx + x\ dy$, where C is the parabola $y = x^2$.

5. Evaluate $\oint_C x^2y^2\ dx - xy^3\ dy$, where C is the triangle with vertices $(0,0), (1,0), (1,1)$.

6. Evaluate $\oint_C (x^2-y^2)\ ds$, where C is the circle $x^2+y^2=4$.

7. Prove Green's Theorem.

8. Evaluate $\oint_C f(x)\ dx + g(y)\ dy$ on any path.

9. Evaluate $\oint_C xy\ dx + (y^2-x)\ dy$, where C is the circle $x^2+y^2=1$.

10. Let $\mathbf{v} = (x+y^2)\mathbf{i} + (y-x^3)\mathbf{j}$ and let C be the ellipse $9x^2+4y^2=36$. Evaluate $\oint_C v_n\ ds$.

11. Evaluate $\int_{(1,1)}^{(3,7)} y\,dx + x\,dy$.

12. Evaluate
$$\int_{(0,1)}^{(\pi/3,e)} (y+\sec^2 x + 2x\log y)\,dx + \left(x+\frac{x^2}{y}\right)\,dy.$$

13. Prove the Test for Independence of Path.

14. Test for Independence of Path, then evaluate $\int_{(1,-2)}^{(3,4)} \frac{y\,dx - x\,dy}{x^2}$ on the line $y = 3x-5$.

15. Test for Independence of Path, then evaluate $\int_{(1,0)}^{(-1,0)} (2xy-1)\ dx + (x^2+6y)\ dy$ on the arc $y = \sqrt{1-x^2}$ for $-1 \le x \le 1$.

16. If C is the circle $x^2+y^2=1$, then evaluate
$$\oint [\sin(xy) + xy\cos(xy)]\ dx + x^2\cos(xy)\ dy.$$

17. Around the square with vertices $(\pm 1, \pm 1)$, evaluate
$$\oint \frac{x^2y\ dx - x^3\ dy}{(x^2+y^2)^2}$$

18. If S is the hemisphere defined parametrically by $x = \sin u \cos v$, $y = \sin u \sin v$, and $z = \cos u$, then evaluate
$$\iint_S dydz + dzdx + dxdy.$$

19. If $\mathbf{w} = xy^2z\mathbf{i} - 2x^3\mathbf{j} + yz^2\mathbf{k}$, S is the surface $1 - x^2 - y^2, x^2+y^2 \le 1$, and $\mathbf{n}$ is the upper normal, then find $\iint_S \mathbf{w}\cdot\mathbf{n}\ ds$.

20. Prove the following theorem: If S is given in the form $z = f(x,y)$ with normal vector $\mathbf{n}$, then
$$\iint_S L\ dy\ dz + M\ dz\ dx + N\ dx\ dy =$$
$$\pm \iint_{R_{xy}} \left(-L\frac{\partial z}{\partial x} - M\frac{\partial z}{\partial y} + N\right)\ dx\ dy$$
where the sign is used when $\mathbf{n}$ is the upper/lower normal.

21. If S is the surface of the region bounded by $z = 1-x^2$, $z = y = 0$, and $y+z=2$, the evaluate
$$\iint_S xy\ dy\ dz + (y^2 + e^{xz^2})\ dz\ dx + \sin(xy)\ dx\ dy.$$

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