1. Let $\mathbf{u} = \mathbf{i} + \mathbf{j} - \mathbf{k}$, $\mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k}$, and $\mathbf{w} = 6\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ be vectors in space.

- **a)** Determine whether $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are linearly independent.
- **b)** Find $||\mathbf{v}||$.
- **c)** Find $||\mathbf{u} \times \mathbf{v}||$.
- **d)** Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.
- **e)** Find $\mathbf{u} \times \mathbf{v} \times \mathbf{w}$.
- **f)** Find the direction cosines of $\mathbf{w}$.
- **g)** Find the area of the parallelogram formed by $\mathbf{u}$ and $\mathbf{w}$.
- **h)** Find the volume of the parallelepiped formed by $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$.

2. Prove that, for any vectors $\mathbf{u}$ and $\mathbf{v}$, $||\mathbf{u} \times \mathbf{v}||^2 = \begin{vmatrix} \mathbf{u}\cdot\mathbf{u} & \mathbf{u}\cdot\mathbf{v} \\ \mathbf{v}\cdot\mathbf{u} & \mathbf{v}\cdot\mathbf{v} \end{vmatrix}$.

3. Using the matrices below, evaluate the expressions that are meaningful.

$$A = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & 7 \end{bmatrix}, \quad H = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}, \quad M = \begin{bmatrix} 2 & -1 & 0 \\ 1 & 2 & 1 \\ 3 & 2 & -1 \end{bmatrix}, \quad N = \begin{bmatrix} 1 & 4 \\ 0 & 3 \\ 7 & 1 \end{bmatrix}$$

a) HM	**c)** $FN + D$	**e)** FNA	**g)** $\det M$	**i)** D^{-1}
b) MH	**d)** $NF - D$	**f)** D^3	**h)** $\det F$	**j)** $D^{-1}A$

4. Using matrix M from Problem 3 and $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$, use Cramer's Rule to solve the system of equations $M\mathbf{x} = \mathbf{b}$. Show your work.

5. Write an equation for:

- **a)** the plane that passes through $(2,1,1)$ and is parallel to the plane $3x - 2y + 5z - 2 = 0$;
- **b)** the line that passes through $(2,-2,3)$ and is perpendicular to the xz-plane;
- **c)** the line that passes through the points $(1,0,3)$ and $(2,-1,4)$.

6. Find all solutions of the system $\begin{cases} x - y + z + w = 0 \\ x + 2y - z - w = 0 \\ 3x - y - z + 2w = 0 \\ x + 3y + z - 2w = 0. \end{cases}$

7. Prove the following.

- **a)** Let A, B, and C be n-square matrices. Prove that if $ABC = I$, then $BCA = I$ and $CAB = I$.
- **b)** Is $A^2 - B^2$ necessarily equal to $(A - B)(A + B)$? When must this be true?

8. Let all matrices in the following be square. Solve for X and Y, stating which matrices are assumed to be nonsingular: $AX + Y = B$, $X + CY = D$.