1. Evaluate the following limits.

(a) $\displaystyle\lim_{\substack{x\to 0\\ y\to 0}} \frac{xy}{x^2+y^2}$

(b) $\displaystyle\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2-y^2}{x^2+y^2}$

(c) $\displaystyle\lim_{\substack{x\to 0\\ y\to 0}} \frac{x^2+\sin^2 y}{y^2}$

(d) $\displaystyle\lim_{\substack{x\to 1\\ y\to 1}} \frac{x-y^4}{x^3-y^4}$

(e) $\displaystyle\lim_{\substack{x\to 0\\ y\to 0}} f(x,y)$ where

$$f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$$

---

2. Describe the regions over which the following functions are continuous.

(a) $f(x,y) = \dfrac{1}{1-x^2-y^2}$

(b) $f(x,y) = \log(x^2-y)$

(c) $f(x,y) = \arctan(x+\sqrt{y})$

(d) $f(x,y,z) = \dfrac{xyz}{x^2+y^2-z}$

(e) $f(x,y) = \begin{cases} \dfrac{3x^2y}{x^2+y^2} & (x,y)\neq(0,0) \\ 0 & (x,y)=(0,0) \end{cases}$

---

3. Consider the relations

$$x = u^2 - v^2, \qquad y = 2uv.$$

Find the Jacobians $\dfrac{\partial(x,y)}{\partial(u,v)}$ and $\dfrac{\partial(u,v)}{\partial(x,y)}$.

---

4. For the polar relations

$$x = r\cos\theta, \qquad y = r\sin\theta,$$

find the Jacobian $\dfrac{\partial(x,y)}{\partial(r,\theta)}$; then find an expression for $\dfrac{\partial(r,\theta)}{\partial(x,y)}$ without computing its Jacobian.

---

5. Find the tangent plane to the surface

$$2x^2 + y^2 + z^2 = 25$$

at the point $(2, \frac{5}{2}\sqrt{2}, \frac{3}{2}\sqrt{2})$.

---

6. Find the tangent line to the curve $x = 2\sqrt{2}\sin t$, $y = 5\cos t$, $z = 3\sin t$ at the point where $t = \frac{\pi}{4}$.

---

7. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for each of the following.

(a) $z = e^{xy}$

(b) $z = x\sin(x+y) - y\cos(x-y)$

(c) $z = \sqrt{x^2-3y}$

(d) $z = \dfrac{e^y}{x^2y^2}$

(e) $z = \arctan(x+\sqrt{y})$

---

8. Find $\dfrac{\partial w}{\partial x}$, $\dfrac{\partial w}{\partial y}$, and $\dfrac{\partial w}{\partial z}$ for $w = f(x,y,z) = \log(x^2+yz+z^2)$.

---

9. Consider the function $z = \dfrac{x}{x+y}$. Find Δz and dz in terms of Δx and Δy at $x=1,\ y=1$. Then compare these two functions for $\Delta x = 0.01$ and $\Delta y = 0.02$.

---

10. (a) Find the Jacobian matrix of

$$y_1 = x_1x_2x_3, \quad y_2 = x_1^2x_3$$

(b) Find the Jacobian matrix of $w = x^2+y^2-z^2$.

(c) Find $\dfrac{\partial(u,v)}{\partial(x,y)}$ for

$$u = x^3 - 3xy^2, \quad v = 3x^2y - y^3$$

---

11. Find $\dfrac{\partial(u,v,w)}{\partial(x,y,z)}$ for

$$u = xe^y\cos z, \quad v = xe^y\sin z, \quad w = xe^y$$

at the point $(\sqrt{2}, \ln 2, \frac{\pi}{4})$.

12. Let $z = x^3 - 3x^2y$ and assume x and y are functions of t such that for $t = 5$ we have $x = 7,\ y = 2,\ \frac{dx}{dt} = 3$, and $\frac{dy}{dt} = -1$. Find $\frac{dz}{dt}$ for $t = 5$.

---

13. Consider the mapping $u = x + y^2z,\ v = x^2y - z,\ w = x^2 + y^2 + z^2$.

(a) Find the Jacobian matrix.

(b) Evaluate the Jacobian at the point $(1,1,2)$.

(c) Does the Jacobian at the point $(1,1,2)$ represent a one-to-one and onto mapping? Explain.

---

14. Consider the functions

$$\begin{aligned} y_1 &= u_2^2 + u_3^2 \\ y_2 &= u_1^2 + u_3^2 \\ y_3 &= u_1^2 + u_2^2 \\ u_1 &= x_1^2 + x_1x_2 \\ u_2 &= x_1^2 + 2x_1x_2 \\ u_3 &= x_1^2 + 3x_1x_2 \end{aligned}$$

Find the Jacobian $\left(\dfrac{\partial y_i}{\partial x_j}\right)$ in the form of a product of two matrices and then evaluate the Jacobian for $x_1 = 1,\ x_2 = 0$.

---

15. Find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$ for

$$x^2 + xu - yv^2 + uv = 1, \quad xu - 2yv = 1$$

at the point where $x = 0,\ y = -1$.