1. Prove the Fundamental Theorem of Calculus.

2. Evaluate the following without a calculator.

(a) $\int_0^{\pi/3} x \sin x \, dx$

(b) $\int_1^{\infty} \frac{\log x}{x} \, dx$

(c) $\int_0^{\sqrt{3}} \arctan x \, dx$

(d) $\int_0^1 \frac{3x}{x^2 - 2x - 8} \, dx$

(e) $\int_0^1 (1 - x^2)^{3/2} \, dx$

3. Evaluate
$$\iint_R 3y \, dx \, dy$$
where R is the semicircle on the half-plane $y \geq 0$.

4. Find the center of mass of the thin plate covering the triangle with vertices $(0,0)$, $(0,1)$, and $(2,0)$, given the density function $f(x,y) = 24x - 12y + 18$.

5. Evaluate
$$\iint_R \mathbf{F}(x,y) \, dx \, dy$$
if R is the triangular region with vertices $(0,0)$, $(1,0)$, and $(0,1)$ and $\mathbf{F}(x,y) = x^2 y \mathbf{i} + x y^2 \mathbf{j}$.

6. Evaluate
$$\iiint_R xyz \, dz \, dy \, dx$$
where R is the unit sphere.

7. Evaluate
$$\iint_{R_{xy}} (1 - x^2 - y^2) \, dx \, dy$$
where R_{xy} is the region $x^2 + y^2 \geq 1$, by converting the integral to polar coordinates.

8. Prove the following theorem.

If $z = f(x,y)$ is defined and has continuous partial derivatives in $R \subseteq D$, then the surface area of a surface in space is
$$S = \iint_R \sqrt{1 + \left(\frac{\partial z}{\partial x}\right)^2 + \left(\frac{\partial z}{\partial y}\right)^2} \, dx \, dy.$$

9. Find the area of the surface defined parametrically by
$$x = u \cos v, \quad y = u \sin v, \quad z = v$$
for $0 \leq u \leq 1$, $0 \leq v \leq 2\pi$.

10. Evaluate
$$\iint_R \frac{1}{x^2 + y^2} \, dx \, dy$$
where region R is the square $|x| < 1$, $|y| < 1$.

11. Prove Leibniz's Rule.

12. Evaluate using Leibniz's Rule:
$$\frac{d}{dx} \int_x^{\tan x} e^{-t^2} \, dt.$$

13. Evaluate
$$\iint_R \frac{\sin y}{y} \, dy \, dx$$
where R is the triangle with vertices at the origin, $(0, \pi)$, and (π, π).

14. Using the substitution $u = 1 + x + y$ and $v = x - y$, evaluate
$$\iint_R \frac{(x-y)^2}{1 + x + y} \, dx \, dy$$
where R is the trapezoid in the first quadrant bounded by the lines $x + y = 2$ and $x + y = 4$.

 Continued on the next page.