Georgia's State-Wide Math Team (Or, All About ARML)

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Outline

All About
ARML

What is
ARML?

What Kinds of
Math Does
ARML
Contain?

How Does a
Student Get
Selected for
ARML?

What Goes
On at ARML
Practice?

How Can I
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Prepared?

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- In 2015, 133 teams participated from 59 regions (that's 1995 students!)
- There is no age or grade restriction; any K-12 student may be part of a team

The ARML Format

Four rounds of competition

- Team Round: 10 problems, 20 minutes
- Power Round: 60 minutes to write proofs
- Individual Round: 10 minutes per pair of problems, 10 problems total
- Relay Round: 6 minutes per 3-person relay, 2 relays

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No calculus; all problems involve "elementary mathematics" (!)

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- In 2015, there were 86 A-division teams and 47 B-division teams

The Georgia ARML Team

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- Others:
  - Debbie Poss (Lassiter, 25 yrs)
  - Carol Sikes (S. Forsyth HS, 7 yrs)
  - Henry Oglesby (Woodstock HS, 5 yrs)
  - Angelique Allen (GAC, 3 yrs)
  - Bobby Stecher (Stratford Academy, 2 yrs)
  - Brian Swanagan (Floyd Co. College & Career Ac., 1 yr)
  - former Georgia ARMLers Harrison Brown, Jonathan Johnson, and Santhosh Karnik (5 yrs each)
  - and me (11 yrs)

The Georgia ARML Team

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- Two teams from 1991 to 2007
- Added a 3rd team in 2008; added a 4th team in 2011 (didn't have a 4th team in 2015)

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  - C team 10th graders and under who will be A or B next year
  - D team Walk-ons and alternates

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    - D team Walk-ons and alternates
- Our A, B, and C teams all compete in Division A

Placements of Georgia ARML

	A division	B division
1989	—	8th
1990	—	4th
1991	9th	8th
1992	1st	11th
1993	3rd	2nd
1994	14th, 25th	—
1995	7th, 23rd	—
1996	9th	13th
1997	13th	3rd
1998	7th, 20th	—
1999	6th, 16th	—
2000	7th	12th
2001	6th	6th
2002	11th	13th

Placements of Georgia ARML

	A division	B division
2003	13th	16th
2004	4th	12th
2005	11th	4th
2006	8th, 19th	—
2007	5th, 30th	—
2008	7th, 29th	72nd
2009	21st, 29th	29th
2010	14th, 47th	16th
2011	10th, 34th	7th, 30th
2012	6th, 56th, 60th	15th
2013	9th, 47th, 53rd	19th
2014	8th, 44th, 53rd	18th
2015	36th, 69th, 76th	—

Best ARML A Teams, Based on Sum of Placements from 2006 to 2015

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1. Lehigh Valley (PA)
2. San Francisco Bay Area
3. New York City
4. North Carolina
5. Southern California
6. Montgomery County (MD)
7. Texas
8. Chicago
9. Georgia
10. Eastern Massachusetts
11. Bergen County (NJ)
12. Washington
13. San Diego
14. Michigan
15. Florida

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Geometry, algebra, analytic geometry, trigonometry, probability logarithms, . . .

What Kinds of Math Does ARML Contain?

Geometry, algebra, analytic geometry, trigonometry, probability
logarithms, . . .
and number theory, combinatorics, sequences, series, vectors,
logic, graph theory, advanced Euclidean geometry, conic
sections, complex numbers, and anything else which isn't
calculus

3. A positive integer has the *Kelly Property* if it contains a zero in its base-17 representation. Compute the number of positive integers less than 1000 (base 10) that have the Kelly Property.

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7. In $\triangle ABC$, point D is on $\overline{AB}$ and point E is on $\overline{AC}$. The measure of the nine angles in triangles ADE, BCD, and CDE can be arranged to form an arithmetic sequence. Compute the greatest possible degree measure of $\angle A$.

Some 2015 Team Round Problems

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8. Let $a_1 = a_2 = a_3 = 1$. For $n > 3$, let a_n be the number of real numbers x such that $x^4 - 2a_{n-1}x^2 + a_{n-2}a_{n-3} = 0$. Compute the sum $a_1 + a_2 + a_3 + \cdots + a_{1000}$.

The 2015 Power Round

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Students must investigate a problem, answer some questions,
make conjectures, and write proofs.
Key idea is sometimes something familiar in disguise

1. The perimeter of regular heptagon *HEPTAGON* is 2015 more than the perimeter of square *ARML*. Let $x = HE - AR$. Compute the greatest possible integer value of x.

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9. An (a, r, m, ℓ)-*trapezoid* is a trapezoid with bases of length a and r, and other sides of length m and ℓ. Compute the number of positive integer values of ℓ such that there exists a $(20, 5, 15, \ell)$-trapezoid.

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10. Six people of different heights are getting in line to buy donuts. Compute the number of ways they can arrange themselves in line such that no three consecutive people are in increasing order of height, from front to back.

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Only 12 out of 1995 participants answered #10 correctly.

Some 2015 Relay Round Problems

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2. Let $T = TNYWR$. A rectangle has area $\frac{T}{4} + \frac{1}{8}$ and a diagonal of length $\frac{T}{2}$. Compute the perimeter of the rectangle.

3. Let $T = TNYWR$, and let k be a real number such that the region above the x-axis satisfying the inequalities $|2x| + |y| \leq 2k$ and $|x| + |y| \geq k$ has area T. Compute k.

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How Does a Student Get Selected for ARML?

- AMC12 and AIME (the USAMO Index)
- KSU Contest
- Individual Varsity tournament results (Georgia only)
- USAMTS
- State Math Tournament is a must
- Middle School and JV kids must WIN a middle school or JV tournament, and have other good scores – better to compete in Varsity

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We must compare unknowns against people we know are good! No selections are based on how well the school or team does at a tournament.

- We select the best 30 we possibly can at the State Tournament (regardless of grade or age) to comprise A and B teams
- We select the best 15 10th graders and younger which weren' already picked for A & B for the C team
- We pick a few alternates and allow "walk-ons" for D team

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"You are 'good' if you are doing what people in the field think is 'good'. Like a good artist is somebody artists think is good, and a good mathematician is somebody mathematicians think is good. The second statement is that 18-year olds are genuinely capable of good work. The third thing is A's ain't it. A's are generally not good work, it represents competent work, and that which is often the antithesis of good work... The missing piece is that 18-year olds can do good work. That's just not in the educational water that people drink. Usually the educational water is 18-year olds can make A's."
–Steve Sigur (1946–2008), GA ARML, 1992–2008

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What Goes On at ARML Practice?

- Schedule:
  - First three Sundays in May, 1:30 to 6:00
  - Memorial Day 10:30 to 6:00
  - Thursday & Friday before ARML, all day
- Practices held at Lassiter, Rockdale, Stratford, Woodstock, and UGA

What Goes On at ARML Practice?

- Practice old ARMLs, particularly Team and Power
- Students teach each other: Georgia ARML veterans help the newbies
- Coaches do not teach (mostly)

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- Until then, our job is to determine who belongs on A, B, and C.
    - A student selected for C may prove themselves at practice and move up to A; or a student selected for A may not be as good as we thought and move down.
    - Who works with the team, who contributes, who keeps focus?
    - Individual ability is slightly less important than ability to work with others

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How Can I Get Students Prepared?

- Take them to as many (Varsity) tournaments as possible
- AMC (both times)
- KSU Contest
- Put books in their hands

Books

- Basics of Math Team: *The Art of Problem Solving*,
  volumes 1 and 2, AoPS

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- Other contest books: ARML, Mandelbrot, etc.

Thank you! Any Questions?

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Website: http://www.drchuckgarner.com

ARML: http://www.arml.com

The Art of Problem Solving: http://www.aops.com