Mathematics Enrichment Through Math Teams (the Georgia Way)

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NAGC Annual Convention
NCSSSMST Professional Conference

Outline

So Why Should I Listen to This Guy?

- 17 years as math teacher
- Sponsor of the Rockdale Magnet Math Team
- GCTM VP for Competitions
- Coach of the Georgia ARML team
- AMC and AIME problem reviewer
- My degrees are all in mathematics, not math ed

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- Curricular problem: If $p(x) = 3x^4 + Ax^3 + x + 3$ and $p(1) = 9$, then find the value of A.
- These are not "problems" they are "exercises"

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- Why do we need to know that the coefficients are nonnegative integers? Why are two values necessary – isn't just $p(10)$ enough? Would it still be solvable if we were given, say, $p(2)$ and $p(5)$?

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Mathematical Topics for Enrichment

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Proofs Induction, contradiction, contrapositive

Connections Geometry and complex numbers; Pythagorean Theorem; binomial coefficients, . . .

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Binomial Coefficients Only moving north and east through lattice points, how many paths exist from the origin to the point $(4,4)$?

Outline

A Personal Perspective

- Math Club
- Math Team
- Mu Alpha Theta

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Meetings! What To Do?

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CONTESTS

- AMC/AIME
- Math League
- Mandelbrot
- Atlantic-Pacific
- American Scholastic Math Association
- National Assessment & Testing
- MAΘ's Log 1 Contest
- Rocket City Math League
- Continental Math League
- Purple Comet
- ARML Local/NYSML Seasonal
- Kennesaw State University Contest

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- high school divided into Junior Varsity and Varsity
  1. Any student who has completed both Algebra II and Geometry is considered Varsity
  2. Under integrated curriculum: Completion of Math III or Accelerated Math II means Varsity
  3. 10th grade and below is JV

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Team round, pair ciphering, individual rounds, etc

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- GCTM Middle School Tournament
- JV State Tournament
- GCTM State Tournament

Outline

The GCTM State Math Tournament

- Begun in 1977; not open to just any school
- Best 36 schools invited to send a team of four
- Teams qualify through any tournament in Georgia
- Qualifying points:
  Number of teams $\div$ Your placement
- Individuals qualify:
  any student who places first

Sample State Tournament Problems

2008 Problem 9

Let $\mathcal{S}$ be the set of all 3-digit positive integers with no repeated digits. A and B are two integers in $\mathcal{S}$ whose digits are prime numbers. C is the largest integer in $\mathcal{S}$. If $A + B = C$, and $A > B$, what is the 3-digit integer A?

2009 Problem 44

A fair coin is tossed multiple times and the results of each toss written in a sequence (i.e., TTHTHTTTH...). If we stop tossing the coin when two consecutive heads appear, what is probability that the sequence of tosses has length 10?

Sample State Tournament Problems

2008 Problem 35

Consider all the segments that cut off a triangle of area A from a given angle. The midpoints of these segments all lie on which type of the following curves?

A) parabola B) circle C) ellipse

D) lemniscate E) hyperbola

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ARML (American Regions Math League)
Georgia team members selected at the State tournament

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To be a member of Georgia ARML is to be one of the best mathematics students in Georgia.

Outline

Rockdale Magnet Math Team

Problems for the Introductory Meeting

Third Week of August, 2009

1. Determine the value of $1 + 2 + 3 + \cdots + 2009 + 2008 + \cdots + 2 + 1$.
2. Choose any positive integer and compute its square. Then add the square to both the original number and the next higher integer. The result will be the next perfect square! Use a picture to explain why this works.
3. Compute the values of 35^2, 45^2, 55^2, and 65^2. What patterns do you notice in the last two digits of the answers? What patterns do you notice in the first two digits?
4. What happens if you add up the first four odd numbers? (In other words, compute $1+3+5+7$.) What happens if you sum the first seven odd numbers? Create pictures to explain the pattern you observe.
5. One of the numbers 212522, 213444, or 214369 is not a perfect square. Identify which one without performing any calculations.
6. Cut a 29×29 square out of one corner of a 71×71 square. Dissect

(1) Rita Rutabega is harvesting turnips. She has a total of 17 different types of turnips in her garden. She wants to pick one of each type to put in her basket. If she can put anywhere from no turnips to 17 turnips in one basket, in how many different ways can Rita fill up her basket?

(2) If there are 31 students in a class, at least how many of them will have birthdays that fall on the same day of the week?

(3) There are 12 students in a class and they must work in groups of four. If Sophia and Amy really want to work together, what is the probability that they are in a group together?

(4) How many six-digit numbers have all their digits of equal parity (i.e. all odd or all even)?

(5) How many three digit numbers do not contain the digits 0 or 9?

(6) Using only the digits 1 and 2, how many 8-digit numbers are there that use each digit at least once?

(7) How many ways are there to put two rooks (castles) of opposite colors on a chessboard so that they cannot attack each other?

(8) Three men arrive at a party wearing hats. They check their hats with the hat-check guy, who is useless. What is the probability that the dumb hat-check guy gives each man the wrong hat?

(9) Four men and four women seat themselves at random around a table. What is the probability that the people are seated alternately by gender?

(10) Do seven-digit decimal numbers with no one's (i.e. the digit 1 appears nowhere in the number) constitute more than 50% of all seven-digit decimal

Rockdale Magnet Math Team

Problems From the Fourth Dimension

First Week of September, 2009

1. Why do we usually refer to a flat surface, such as a sheet of paper or a markerboard, as a two-dimensional space?

2. How many quantities are needed to specify a single colored pixel on a standard computer monitor?

3. In order to find the distance between two points in four-dimensional space, one must first understand distance in two or three dimensions. To begin, what is the distance between the points $(2006, 2007)$ and $(2010, 2002)$? In general, how do we compute distance in the plane?

4. Now figure out how far apart the "prime" points $(2, 5, 11)$ and $(3, 7, 13)$ are in three dimensions. Also calulate the distance from the origin $(0, 0, 0)$ to the point $(1, \frac{1}{2}, \frac{1}{3})$.

5. Demonstrate your mastery of the fourth dimension by extending the ideas begun in the previous two problems to predict the distance be-

2.3 Uncommon Residues

The solutions to these problems involve residues. An integer a is a residue of integer b modulo m if and only if $a \equiv b \bmod m$; in other words, if and only if m divides $a - b$. We have seen simple congruences before; the congruences required here are more subtle. We begin with a simple example involving Euler's Theorem to get you started.

Example 2.12. *Find the last two digits of the number* 1707^{365}.

Solution. This is equivalent to finding the residue of 1707^{365} mod 100. This reminds us of Euler's Theorem: For relatively prime integers a and m, we have that $a^{\phi(m)} \equiv 1 \bmod m$, where $\phi(m)$ denotes the number of positive integers less than m and relatively prime to m.

Since $\phi(100) = 40$, we have that $1707^{40} \equiv 1 \bmod 100$. Thus,

$$1707^{365} \equiv (1707^{40})^9 1707^5 \equiv (1)^9 1707^5 \equiv 1707^5 \equiv 7^5 \equiv 7^4 7 \equiv 2401 \cdot 7 \equiv 7 \bmod 100,$$

which implies the last two digits are 07. □

Other facts are useful in digits problems. The most notable is that the number of digits needed to express an integer k is $\lfloor \log k \rfloor$, where $\lfloor x \rfloor$ is the greatest integer, or 'floor', function.

Example 2.13. *Denote by $S(m)$ the sum of the digits of the positive integer m. Prove that there does not exist a number N such that $S(2^n) \leq S(2^{n+1})$ for all $n \geq N$.* Quantum (Russia)

Solution. The problem deals with the sum of the digits. We know the divisibility rule for 9: if the sum of the digits is divisible by 9, then the number is divisible by 9. This leads to the idea of

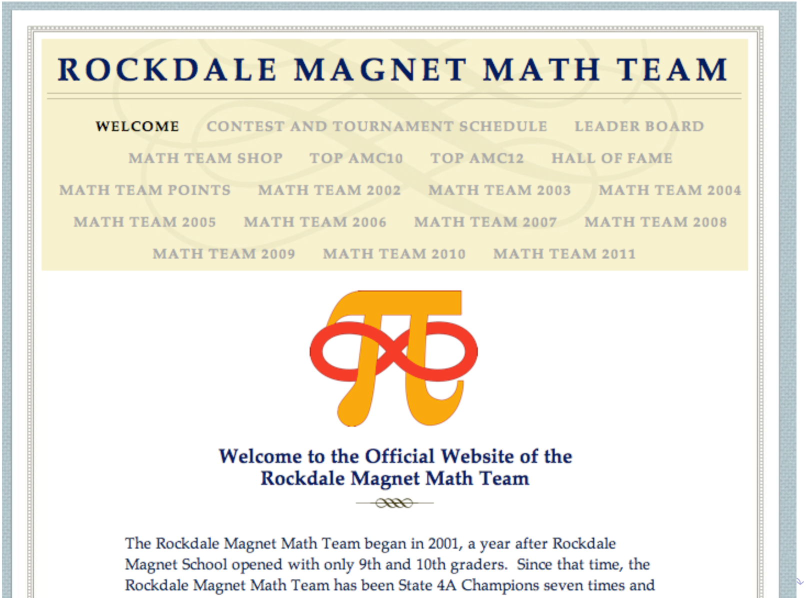
Branding

Rockdale Magnet

MATH TEAM

Apparel, signs, banners, websites, printed materials

web.me.com/drcgarner/MathTeam


ROCKDALE MAGNET MATH TEAM

WELCOME | CONTEST AND TOURNAMENT SCHEDULE | LEADER BOARD | MATH TEAM SHOP | TOP AMC10 | TOP AMC12 | HALL OF FAME | MATH TEAM POINTS | MATH TEAM 2002 | MATH TEAM 2003 | MATH TEAM 2004 | MATH TEAM 2005 | MATH TEAM 2006 | MATH TEAM 2007 | MATH TEAM 2008 | MATH TEAM 2009 | MATH TEAM 2010 | MATH TEAM 2011

Welcome to the Official Website of the Rockdale Magnet Math Team

The Rockdale Magnet Math Team began in 2001, a year after Rockdale Magnet School opened with only 9th and 10th graders. Since that time, the Rockdale Magnet Math Team has been State 4A Champions seven times and

Outline

Books

- *The Art of Problem Solving*, volume 1 and 2
- *Art and Craft of Problem Solving*, Paul Zeitz
- *Contest Problem Books* (AMC), volumes I–IX, MAA
- *First Steps for Math Olympians*, J. Douglas Faires
- *100+ Problems in...* series, Titu Andreescu
- *Problem Solving Strategies*, George Engel
- Other contest books: ARML, Mandelbrot, etc.

Web Resources

- http://www.artofproblemsolving.com
- http://www.mathcircles.org
- http://www.mathteacherscircle.org
- http://www.gctm.org

Thank you!

This presentation, with active hyperlinks, will be posted at

`http://web.me.com/drcgarner`

(Click on "Presentations" at the top)