

MATH EDUCATION IN GEORGIA REFLECTED THROUGH THE STATE MATH TOURNAMENT

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OUTLINE

- 1 INTRODUCTION
- 2 TOPIC BREAKDOWN
- 3 SOME TOPICS DON'T CHANGE
- 4 SOME TOPICS DISAPPEAR
- 5 SOME TOPICS EMERGE
- 6 EMPHASIS FROM RULES TO PROBLEM SOLVING
- 7 DIFFICULTY COMPARISON
- 8 CONCLUSION

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TOURNAMENT THEN

- First proposed in 1972, began in 1977
- Four-student teams
- 50-problem multiple-choice test,
Scoring: $50 + 4C - I$
- 10 individual ciphering problems
- No calculators
- Individual winners determined solely by test
- Total ciphering and test scores determined winning teams

TOURNAMENT CHANGES

- 1993: Individual ciphering and test determined individual winners
- 1994: Calculators allowed for the first time
- 1995: 45-problem multiple-choice and 5 free-response,
Scoring became $50 + 4C - I_{mc}$
- 1995: Pair ciphering introduced
- 2008: Scoring became $5C + B$

TOURNAMENT NOW

- Four-student teams
- 45-problem multiple-choice plus 5 free-response test, with calculators
- 10 individual ciphering problems
- 8 pair ciphering problems
- Individual winners determined solely by test and ciphering
- Individual scores and pair ciphering determine team winners

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TOPIC BREAKDOWN BY NUMBER OF PROBLEMS

	1988	2009
Algebra	17	10-18
Geometry	10	8-12
Analytic Geometry	5	5-10
Trigonometry	8	4-8
Calculus	3	1-3
Analysis	0	4-8
Discrete math	7	10-15

“Discrete” includes counting, probability, statistics, logic, sequences, series, and number theory.

OUTLINE

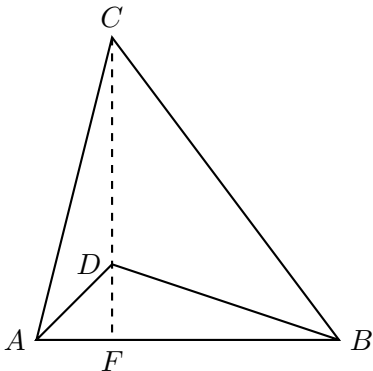
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WE STILL SEE GEOMETRY

1984 Problem 16

If D is between C and F ,
 $\overline{CF} \perp \overline{AB}$, $CD = 3$, and the area
of quadrilateral $ADBC$ is 5, then
what is the length of \overline{AB} ?

- A) 6
 B) $\frac{10}{3}$
 C) 5
 D) $\frac{9}{4}$
 E) 4

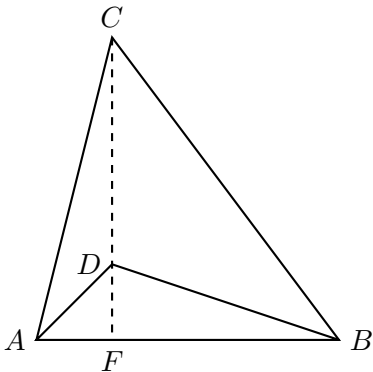


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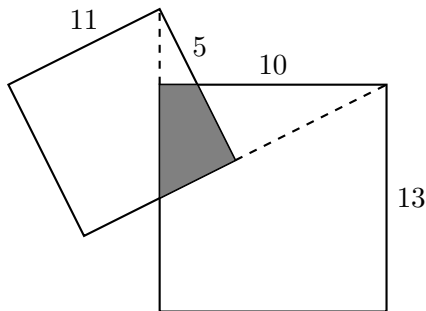


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1984 Problem 27

Given the intersecting squares with lengths as indicated in the figure at left, compute the area of the shaded lozenge.

- A) $315/8$
- B) 39
- C) $307/8$
- D) $507/8$
- E) $363/8$

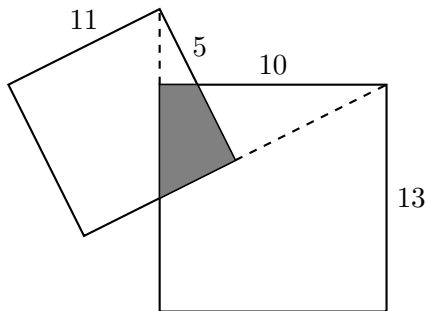


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WE STILL SEE ALGEBRA

1987 Problem 15

The solution of the equation

$$\frac{\sqrt{x+1} + \sqrt{x-1}}{\sqrt{x+1} - \sqrt{x-1}} = 3$$

is

- A) $4/5$
- B) $5/4$
- C) $3/5$
- D) $5/3$
- E) 3

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Note the emphasis on algebraic manipulation.

WE STILL SEE ALGEBRA

2007 Problem 20

When $(1 - 2x)^3(1 + kx)^2$ is expanded, two values of k give the coefficient of x^2 as 30. The sum of these two values of k is

- A) -1
- B) 8
- C) 10
- D) 12
- E) 14

WE STILL SEE ALGEBRA

2007 Problem 20

When $(1 - 2x)^3(1 + kx)^2$ is expanded, two values of k give the coefficient of x^2 as 30. The sum of these two values of k is

- A) -1
- B) 8
- C) 10
- D) 12
- E) 14

Now algebra is embedded in other problems.

WE SEE “DEFINED OPERATIONS” PROBLEMS

1999 Problem 11

The operation \otimes is defined by $a \otimes b = \log_b a$. Then $(5 \otimes 25) \otimes 2 =$
A) -2 B) -1 C) 1 D) 2 E) None of these

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2007 Problem 18

The relation n^* is defined for whole numbers as follows: $0^* = 0$, $1^* = 1$, and $n^* = n \cdot (n - 2)^*$ for $n \geq 2$. Evaluate the expression

$$\frac{25^*}{21^* - 20^*}.$$

- A) 0 B) 575 C) $12,075$ D) $7,905,853,580,625$
 E) The expression is undefined.

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WE NO LONGER SEE GRAPHING PROBLEMS...

1984 Problem 24

Which of these is the polar equation for a line?

A) $\theta = \pi/4$

B) $r = 5$

C) $r = 2\theta$

D) $r = 4 \cos \theta$

E) $r = 4 \sin \theta$

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1984 Problem 24

Which of these is the polar equation for a line?

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B) $r = 5$

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D) $r = 4 \cos \theta$

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...BUT WE STILL SEE PROBLEMS ABOUT GRAPHS

2008 Problem 35

Consider all the segments that cut off a triangle of area A from a given angle. The midpoints of these segments all lie on which type of the following curves?

- A) parabola
- B) circle
- C) ellipse
- D) lemniscate
- E) hyperbola

...BUT WE STILL SEE PROBLEMS ABOUT GRAPHS

2008 Problem 35

Consider all the segments that cut off a triangle of area A from a given angle. The midpoints of these segments all lie on which type of the following curves?

- A) parabola
- B) circle
- C) ellipse
- D) lemniscate
- E) hyperbola

WE NO LONGER SEE PROGRAMMING...

1988 Problem 33

What will be printed during the execution of this BASIC language program?

```
10 S=0
20 FOR I=1 TO 200
30 S=S+I
40 NEXT I
50 PRINT S
```

- A) 200
- B) 4000
- C) 5050
- D) 20100
- E) None of these

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1988 Problem 33

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- B) 4000
- C) 5050
- D) 20100
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...BUT WE STILL SEE LOGIC

2008 Problem 15

Andre is a butcher and president of the street storekeepers' committee, which also includes a grocer, a baker, and a florist. All of them sit around a table. Andre sits on Charmeil's left. Berton sits at the grocer's right. Duclos, who faces Charmeil, is not the baker. What occupation does Berton have?

- A) butcher
- B) baker
- C) grocer
- D) florist
- E) The answer cannot be determined from the information given

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NUMBER THEORY

2008 Problem 7

If $210_{10} = 420_n$, where the subscripts indicate the base, then what is 121_n in base 10?

- A) 36
- B) 64
- C) 81
- D) 121
- E) 256

NUMBER THEORY

2008 Problem 7

If $210_{10} = 420_n$, where the subscripts indicate the base, then what is 121_n in base 10?

- A) 36
- B) 64
- C) 81
- D) 121
- E) 256

NUMBER THEORY

2008 Problem 9

Let \mathcal{Q} be the set of all 3-digit positive integers with no repeated digits. A and B are two integers in \mathcal{Q} whose digits are prime numbers. C is the largest integer in \mathcal{Q} . If $A + B = C$, and $A > B$, what is the 3-digit integer A ?

- A) 527
- B) 571
- C) 735
- D) 752
- E) 923

NUMBER THEORY

2008 Problem 9

Let \mathcal{Q} be the set of all 3-digit positive integers with no repeated digits. A and B are two integers in \mathcal{Q} whose digits are prime numbers. C is the largest integer in \mathcal{Q} . If $A + B = C$, and $A > B$, what is the 3-digit integer A ?

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- B) 571
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NUMBER THEORY

2008 Problem 23

What are the last three digits of 7^{9999} ?

- A) 143
- B) 343
- C) 543
- D) 743
- E) 943

NUMBER THEORY

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DEMOIVRE'S THEOREM (RULE)

1984 Problem 8

DeMoivre's Theorem predicts that $[2(\cos \theta + i \sin \theta)]^3$ is

- A) $8(\cos 3\theta + i \sin 3\theta)$
- B) $8(\cos^3 \theta + i \sin^3 \theta)$
- C) $6(\cos^3 \theta + i \sin^3 \theta)$
- D) $6(\cos 3\theta + i \sin 3\theta)$
- E) $8(\cos^3 \theta - i \sin^3 \theta)$

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DEMOIVRE'S THEOREM (PROBLEM SOLVING)

2009 Problem 28

Let z be a complex root of $z^6 + z^3 + 1 = 0$. Then z^{2009} must be equal to

- A) 1
- B) z
- C) z^2
- D) z^3
- E) z^4

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TRIGONOMETRIC IDENTITIES (RULE)

1984 Problem 15

Determine which of the following is *not* an identity.

A) $\frac{(1 - \sin^2 x)^{3/2}}{\sec x} = \cos 2x + \sin^4 x$

B) $\tan(x + 45^\circ) = \frac{1 + \tan x}{1 - \tan x}$

C) $\frac{\cos x}{1 + \sin x} = \sec x - \tan x$

D) $\cot x \csc x - 2 = 2 \cot x - \csc x$

E) $\sin(x - 75^\circ) \cos(x + 75^\circ) - \cos(x - 75^\circ) \sin(x + 75^\circ) = -\frac{1}{2}$

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Two scientists stand on perfectly level ground d feet apart. One scientist shines a light beam into the sky at an angle of elevation of α degrees, and the other shines a light beam into the sky at an angle of elevation of β degrees. Given that their beams do intersect and that $0 < \alpha, \beta < \frac{\pi}{2}$, how many feet above they ground will they do so?

- A) $\frac{d \sin \alpha \sin \beta}{\sin(\alpha + \beta)}$
 B) $\frac{d \sin \alpha \sin \beta}{\sin(\alpha - \beta)}$
 C) $\frac{d \tan \alpha \tan \beta}{\tan(\alpha - \beta)}$
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 E) None of these

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E) None of these

SYSTEM OF EQUATIONS (RULE)

1986 Problem 3

If (x, y) is the ordered pair of real numbers satisfying the matrix equation

$$\begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 7 \end{bmatrix}$$

determine the value of $x - y$.

- A) -19
- B) 1
- C) 3
- D) 5
- E) 13

SYSTEM OF EQUATIONS (RULE)

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- A) -19
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SYSTEM OF EQUATIONS (PROBLEM SOLVING)

2007 Problem 1

Let a be a positive integer. Given the system of equations below, determine the maximum possible value of $x + y + z$.

$$2x + a = y$$

$$a + y = x$$

$$x + y = z$$

- A) -10
- B) -6
- C) $-14/3$
- D) -2
- E) 0

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MATRICES (RULE)

1990 Problem 42

Solve for x :

$$\begin{vmatrix} 3 & 4 & x \\ 0 & 1 & 0 \\ 2 & 5 & 6 \end{vmatrix} = 2$$

- A) -3
- B) 0
- C) 2
- D) 3
- E) 8

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SYSTEM OF EQUATIONS (PROBLEM SOLVING)

2005 Problem 16

Find x if

$$\begin{vmatrix} x-1 & x & x+1 \\ x & x+2 & 1 \\ 0 & x & 0 \end{vmatrix} = -x.$$

- A) 0
- B) $-i, i$
- C) $-i\sqrt{2}, i\sqrt{2}, 0$
- D) $-i\sqrt{2}, i\sqrt{2}$
- E) $-i\sqrt{2}, -i, i, i\sqrt{2}, 0$

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COMPLEX NUMBERS (RULE)

1990 Problem 42

Which of the following could be a solution to $x^6 = -i$?

A) $\frac{\sqrt{3}}{2} + \frac{1}{2}i$

B) $\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

C) $\frac{1}{2} + \frac{\sqrt{3}}{2}i$

D) $-\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i$

E) $-i$

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COMPLEX NUMBERS (PROBLEM SOLVING)

2007 Problem 27

Professor Cal Q. Luss was sitting at the circular dining table in the lounge, when two of his colleagues, Professors Al G. Brah and Stat S. Tix, joined him at the table. Professor Luss exclaimed: "If the table has a radius of 1 unit, and Professor Brah is assigned the complex number z , then Professor Tix has the value $z + 1$! This means that both of you are n th roots of 1!" What is the value of n ?

- A) 3
- B) 4
- C) 5
- D) 6
- E) 8

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DIFFICULT PROBABILITY

1984 Problem 42

Two balls are drawn at random at the same time from a bowl containing 3 black balls and 8 white ones. What is the probability that both balls are white?

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Answer: $28/55$

2009 Problem 44

A fair coin is tossed multiple times and the results of each toss written in a sequence (i.e., THTHTTTH...). If we stop tossing the coin when two consecutive heads appear, what is probability that the sequence of tosses has length 10?

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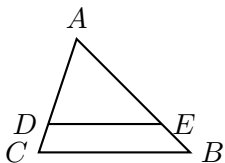
2009 Problem 44

A fair coin is tossed multiple times and the results of each toss written in a sequence (i.e., THTHTTTTH...). If we stop tossing the coin when two consecutive heads appear, what is probability that the sequence of tosses has length 10?

Answer: $17/512$

DIFFICULT GEOMETRY

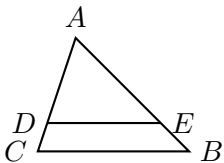
1984 Problem 47



In the figure, $AC = 8$, $CB = 6$, $AB = 10$, $\overline{DE} \parallel \overline{CB}$ and the area of $\triangle ADE$ equals one-half the area of $\triangle ABC$. Then $DC = ?$

DIFFICULT GEOMETRY

1984 Problem 47

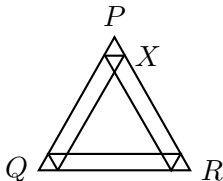


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Answer: $8 - 4\sqrt{2}$

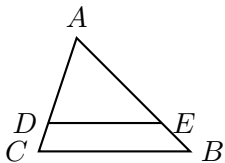
2009 Problem 43

In the figure, all segments are parallel to one of the sides of the equilateral triangle PQR which has side length 1. How long should PX be to maximize the smallest of the ten areas defined?



DIFFICULT GEOMETRY

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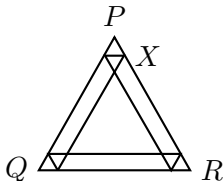


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Answer: $8 - 4\sqrt{2}$

2009 Problem 43

In the figure, all segments are parallel to one of the sides of the equilateral triangle PQR which has side length 1. How long should PX be to maximize the smallest of the ten areas defined?



Answer: $1/4$

DIFFICULT CONICS

1988 Problem 47

Write the equation that describes the set of all points (x, y) that are equidistant from the x -axis and the point $(4, 6)$.

DIFFICULT CONICS

1988 Problem 47

Write the equation that describes the set of all points (x, y) that are equidistant from the x -axis and the point $(4, 6)$.

Answer: $(x - 4)^2 = 12(y - 3)$

2006 Problem 43

Consider the conic

$$\frac{(x - 7)^2}{8} + \frac{(y + 3)^2}{9} = 1.$$

If P is an endpoint of the minor axis and K is a focal point, then find the length of \overline{PK} .

DIFFICULT CONICS

1988 Problem 47

Write the equation that describes the set of all points (x, y) that are equidistant from the x -axis and the point $(4, 6)$.

Answer: $(x - 4)^2 = 12(y - 3)$

2006 Problem 43

Consider the conic

$$\frac{(x - 7)^2}{8} + \frac{(y + 3)^2}{9} = 1.$$

If P is an endpoint of the minor axis and K is a focal point, then find the length of \overline{PK} .

Answer: 3

“DIFFICULT” CONICS?

1990 Problem 6

Find the distance between the centers of the circles

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2006 Problem 22

The shortest distance between a point on circle $x^2 + y^2 - 9 = 0$ and on circle $x^2 + y^2 - 12x + 6y + 41 = 0$ is ?

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A bag contains pennies, nickels, dimes, and quarters. There are twice as many nickels as pennies, half as many quarters as dimes, and three times as many dimes as pennies. Which of the following could be the amount of money in the bag?

- A) \$7.65 B) \$7.70 C) \$7.75 D) \$7.80 E) \$7.85

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OUTLINE

- 1 INTRODUCTION
- 2 TOPIC BREAKDOWN
- 3 SOME TOPICS DON'T CHANGE
- 4 SOME TOPICS DISAPPEAR
- 5 SOME TOPICS EMERGE
- 6 EMPHASIS FROM RULES TO PROBLEM SOLVING
- 7 DIFFICULTY COMPARISON
- 8 CONCLUSION

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Thank You!