Table Problems in Calculus

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DOE AP Regional Workshop, Lambert High School,
October 23, 2015

Outline

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Single Question Problems

Multiple Question Problems

1. The table shows the velocity of a vintage sports car accelerating from 0 to 142 miles per hour in 36 seconds (0.01 hours). Use a Riemann sum to estimate how far the car traveled during the 36 seconds it took to reach 142 mph. Roughly how many seconds did it take the car to reach the halfway point? About how fast was the car going then?

Hours	MPH
0.000	0
0.001	40
0.002	62
0.003	82
0.004	96
0.005	108
0.006	116
0.007	125
0.008	132
0.009	137
0.010	142

2. In order to determine the average temperature for the day, a meteorologist decides to record the temperature at eight times. She decides that these recordings do not have to be equally spaced during the day because she does not need to make several readings during those periods when the temperature is not changing much. She makes one reading at some time during each of the intervals in the table. Calculate the average temperature.

Time	Temp
12AM-5AM	42°
5AM-7AM	57°
7AM-9AM	72°
9AM-1PM	84°
1PM-4PM	89°
4PM-7PM	75°
7PM-9PM	66°
9PM-12AM	52°

4. The values of five different continuous and differentiable functions are given in the table below.

x	$p(x)$	$q(x)$	$r(x)$	$s(x)$	$t(x)$
2	-2	-8	-2	3	0
4	0	-12	4	5	2
6	2	-18	8	8	6
7	4	-26	14	9	11

From the information given in the table, which functions could be concave down for $2 < x < 7$? Justify your answer.

5. The table below gives selected values of f, f', g, and g'.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	-4	9	-1
2	-5	6	2	5
4	1	3	7	8

Let $h(x) = f(x)g(x)$. Compute $h'(2)$.

7. The function f is continuous on the interval $[0, 2]$ and has values that are given in the table below.

x	0	1	2
$f(x)$	-1	k	-2

Find all possible values of k so that $f(x)$ has at least two zeros in the interval $[0, 2]$.

9. The data in the table below represents the rate of light output of a strobe light at a given time.

Time (milliseconds)	0	5	10	15	20	25	30
Output (millions of lumens)	0	0.2	0.6	2.6	4.2	3	1.8

Using a midpoint approximation with three equal subintervals, estimate the total light output of the bulb over the 30 milliseconds, measured in million lumen-milliseconds.

11. The table below gives data points for a continuous function $f(x)$ for $3 \leq x \leq 12$.

x	3	5	8	12
$f(x)$	2	4	6	4

Let L be the left Riemann sum on three subintervals which approximates $\int_3^{12} f(x)\, dx$. Let R be the right Riemann sum on three subintervals which approximates $\int_3^{12} f(x)\, dx$. Compute $|L - R|$.

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12. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and at $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Evaluate the first derivatives of the following combinations of f and g at the given value of x.

(a) $3f(x) - g(x), \quad x = -1$

(b) $f^3(x)g^3(x), \quad x = 0$

(c) $g(f(x)), \quad x = -1$

(d) $f(g(x)), \quad x = -1$

(e) $\dfrac{f(x)}{g(x)+2}, \quad x = 0$

(f) $g(x + f(x)), \quad x = 0$

15. The table below shows temperature readings of a cup of coffee which was placed on a counter and left to cool. The differentiable function $C(t)$ models the temperature of the coffee for $0 \leq t \leq 10$.

t (minutes)	0	3	4	6	8	9	10
$C(t)$ (°F)	180	175	172	168	164	162	161

(a) Find the average rate of change of the temperature for $0 \leq t \leq 10$.

(b) Must there be a time t, for $0 < t < 10$, where $C'(t)$ equals the value found in part (a)? Justify your answer.

(c) Estimate $C'(8)$.

17. The rate at which water flows out of a pipe, in gallons per minute, is given by a differentiable function W of time t. The table below shows the rate as measured every 5 minutes for a 40-minute period.

t	0	5	10	15	20	25	30	35	40
W	9.5	10.4	10.8	11.3	11.7	11.2	10.7	10.1	9.6

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_0^{40} W(t)\ dt$. Explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 40$, such that $W'(t) = 0$?

(c) The rate of the water flow $W(t)$ can be approximated by $X(t) = \frac{1}{210}\left(2001 + 39t - t^2\right)$. Use $X(t)$ to approximate the average rate of water flow during the 40-minute time period. Indicate units of measure.

18. Consider the table below, showing values for the differentiable function f.

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	5.0	3.5	2.6	2.0	1.5

(a) Estimate $f'(1.4)$.

(b) Give an equation for the tangent line to the graph of f at $x = 1.4$.

(c) Is $f''(x)$ positive, negative, or zero? Explain how you determine this.

(d) Using the data in the table, find a midpoint approximation with 2 equal subintervals for $\int_{1.0}^{1.8} f(x)\ dx$.

19. A racehorse is running a 10 furlong race (1 furlong is 220 yards). As the horse passes each furlong marker, F, a steward records the time elapsed, t, since the beginning of the race, as shown in the table below.

t	0	20	33	46	59	73	86	100	112	124	135
F	0	1	2	3	4	5	6	7	8	9	10

(a) How long does it take the horse to finish the race?

(b) What is the average speed of the horse over the the first 5 furlongs?

(c) What is the approximate speed of the horse as it passes the 3-furlong marker?

(d) During which portion of the race is the horse running the fastest? Accelerating the fastest?

23. The table below shows some values for a twice-differentiable function g.

x	-1	0	1	2	3	4	5
$g(x)$	8	5	-2	1	-1	-3	3

(a) Approximate $\int_{-1}^{5} g(x)\,dx$ using a left Riemann sum over 3 subintervals of equal length.

(b) Using the aprroximation from part (a), estimate the average value of g over $[-1, 5]$.

(c) Find the average rate of change of g over $[-1, 5]$.

(d) For $-1 < q < 1$, explain why there must exist q such that $g(q) = 0$.

(e) For $-1 < r < 1$, explain why there must exist r such that $g'(r) = -5$.

24. The functions g and h are differentiable for all real numbers. The table below gives values of g and h and their first derivatives. The function f is given by $f(x) = 2g(h(x)) - 17$.

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
0	5	1	6	-3
1	8	3	5	-1
2	11	6	-1	1
3	-2	2	-3	4

(a) For $0 < c < 3$, must there be a value of c such that $f(c) = 2$? Justify your answer.

(b) For $0 < d < 3$, must there be a value of d such that $f'(d) = 2$? Justify your answer.

(c) Using values as given in the table, compute $f'(3)$.

26. An object moves in a straight line for 12 seconds. The velocity, in meters per second, and the acceleration, in meters per second per second, are differentiable functions. The table shows selected values of the functions.

t (sec)	0	2	5	6	10	12
$v(t)$ (m/s)	0	4	8	6	10	30
$a(t)$ (m/s^2)	1	3	2	4	1	3

(a) Using the subintervals as indicated in the table, use a trapezoidal sum to estimate $\int_0^{12} v(t)\ dt$.

(b) Explain the meaning of $\frac{1}{12}\int_0^{12} v(t)\ dt$ using correct units.

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$a(t)$ (m/s^2)	1	3	2	4	1	3

(c) Approximate $\int_0^{12} a(t)\ dt$ using a trapezoid sum with subintervals as indicated in the table. What is the actual value of $\int_0^{12} a(t)\ dt$?

(d) Explain why acceleration must equal $\frac{1}{2}$ for some value of t in the interval $2 \leq t \leq 6$.

29. The table below shows four points on the differentiable function $f(t)$.

t	$f(t)$
8	8.253
9	8.372
10	8.459
11	8.616

(a) Estimate $f'(10)$.

(b) Estimate $f^{-1}(8.5)$

(c) Estimate $\int_8^{10} f'(t)\ dt$.

(d) Estimate $\left(f^{-1}\right)'(8.4)$.

Thanks!

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