Table Problems in Calculus

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1 Single Question Problems

1. The table shows the velocity of a vintage sports car accelerating from 0 to 142 miles per hour in 36 seconds (0.01 hours). Use a Riemann sum to estimate how far the car traveled during the 36 seconds it took to reach 142 mph. Roughly how many seconds did it take the car to reach the halfway point? About how fast was the car going then?

Hours	MPH
0.000	0
0.001	40
0.002	62
0.003	82
0.004	96
0.005	108
0.006	116
0.007	125
0.008	132
0.009	137
0.010	142

2. In order to determine the average temperature for the day, a meteorologist decides to record the temperature at eight times. She decides that these recordings do not have to be equally spaced during the day because she does not need to make several readings during those periods when the temperature is not changing much. She makes one reading at some time during each of the intervals in the table. Calculate the average temperature.

Time	Temp
12AM-5AM	42°
5AM-7AM	57°
7AM-9AM	72°
9AM-1PM	84°
1PM-4PM	89°
4PM-7PM	75°
7PM-9PM	66°
9PM-12AM	52°

3. Assume the following function f is a decreasing continuous function on the interval $[0, 4]$ and that the following is a table showing some function values.

x	0	1	1.5	3	4
$f(x)$	4	3	2	1.5	1

Use a trapezoid sum to approximate the area under f.

4. The values of five different continuous and differentiable functions are given in the table below.

x	$p(x)$	$q(x)$	$r(x)$	$s(x)$	$t(x)$
2	-2	-8	-2	3	0
4	0	-12	4	5	2
6	2	-18	8	8	6
7	4	-26	14	9	11

From the information given in the table, which functions could be concave down for $2 < x < 7$? Justify your answer.

5. The table below gives selected values of f, f', g, and g'.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
0	3	-4	9	-1
2	-5	6	2	5
4	1	3	7	8

Let $h(x) = f(x)g(x)$. Compute $h'(2)$.

6. The table below gives some values of the function $f(x)$, which is continuous for $0 \le x \le 5$ and differentiable for $0 < x < 5$.

x	0	1	2	3	4	5
$f(x)$	3	5	4	1	3	4

Using the values in the table, estimate $f'(3)$.

7. The function f is continuous on the interval $[0, 2]$ and has values that are given in the table below.

x	0	1	2
$f(x)$	-1	k	-2

Find all possible values of k so that $f(x)$ has at least two zeros in the interval $[0, 2]$.

8. A radar gun was used to record the velocity, in meters per second, of a runner. These velocities are shown in the table below.

t (sec)	0	4.3	6.5	8
v (m/sec)	0	10.83	12.46	9.32

Use a trapezoid sum to approximate the distance the runner traveled in 8 seconds.

9. The data in the table below represents the rate of light output of a strobe light at a given time.

Time (milliseconds)	0	5	10	15	20	25	30
Output (millions of lumens)	0	0.2	0.6	2.6	4.2	3	1.8

Using a midpoint approximation with three equal subintervals, estimate the total light output of the bulb over the 30 milliseconds, measured in million lumen-milliseconds.

10. The table below gives data points for a continuous function $f(x)$ for $2 \leq x \leq 10$.

x	2	5	7	8	10
$f(x)$	3	1	5	3	5

Using subintervals as indicated by the table, estimate $\int_2^{10} f(x)\, dx$ with a trapezoidal approximation.

11. The table below gives data points for a continuous function $f(x)$ for $3 \leq x \leq 12$.

x	3	5	8	12
$f(x)$	2	4	6	4

Let L be the left Riemann sum on three subintervals which approximates $\int_3^{12} f(x)\, dx$. Let R be the right Riemann sum on three subintervals which approximates $\int_3^{12} f(x)\, dx$. Compute $|L - R|$.

2 Multiple Question Problems

12. Suppose that functions f and g and their first derivatives have the following values at $x = -1$ and at $x = 0$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	0	-1	2	1
0	-1	-3	-2	4

Evaluate the first derivatives of the following combinations of f and g at the given value of x.

(a) $3f(x) - g(x), \quad x = -1$

(b) $f^3(x)g^3(x), \quad x = 0$

(c) $g(f(x)), \quad x = -1$

(d) $f(g(x)), \quad x = -1$

(e) $\dfrac{f(x)}{g(x)+2}, \quad x = 0$

(f) $g(x + f(x)), \quad x = 0$

13. Suppose that functions $f(x)$ and $g(x)$ and their first derivatives have the following values at $x = 0$ and $x = 1$.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
0	1	1	-3	$\frac{1}{2}$
1	3	5	$\frac{1}{2}$	-4

Find the first derivatives of the following combinations at the given value of x.

(a) $6f(x) - g(x)$ at $x = 1$

(b) $f(x)g^2(x)$ at $x = 0$

(c) $\dfrac{f(x)}{g(x)+1}$ at $x = 1$

(d) $f(g(x))$ at $x = 0$

(e) $g(f(x))$ at $x = 0$

(f) $(x + f(x))^{3/2}$ at $x = 1$

(g) $f(x + g(x))$ at $x = 0$

14. The function $f(x)$ has values given in the table. Estimate the following values.

(a) $f'(4)$

(b) $f'(5)$

x	$f(x)$
3	1
3.86	1.27
3.98	1.88
4	2
4.05	2.05
4.1	2.35
4.9	1.21
4.98	1.1
5	1
5.02	0.95
5.05	0.9
6	0

15. The table below shows temperature readings of a cup of coffee which was placed on a counter and left to cool. The differentiable function $C(t)$ models the temperature of the coffee for $0 \leq t \leq 10$.

t (minutes)	0	3	4	6	8	9	10
$C(t)$ (°F)	180	175	172	168	164	162	161

(a) Find the average rate of change of the temperature for $0 \leq t \leq 10$.

(b) Must there be a time t, for $0 < t < 10$, where $C'(t)$ equals the value found in part (a)? Justify your answer.

(c) Estimate $C'(8)$.

16. Oil is leaking out of a tanker damaged at sea. The damage to the tanker is worsening as evidenced by the increased leakage each hour, recorded in the following table.

Time (hrs)	0	1	2	3	4	5	6	7	8
Leakage (gal/hr)	50	70	97	136	190	265	369	516	720

(a) Give an upper and lower estimate of the total quantity of oil that has escaped after 8 hours.

(b) The tanker continues to leak 720 gal/hr after the first 8 hours. If the tanker originally contained 25,000 gallons of oil, approximately how many more hours will elapse in the worst case before all the oil has spilled? In the best case?

17. The rate at which water flows out of a pipe, in gallons per minute, is given by a differentiable function W of time t. The table below shows the rate as measured every 5 minutes for a 40-minute period.

t	0	5	10	15	20	25	30	35	40
W	9.5	10.4	10.8	11.3	11.7	11.2	10.7	10.1	9.6

(a) Use a midpoint Riemann sum with 4 subdivisions of equal length to approximate the value of $\int_0^{40} W(t)\,dt$. Explain the meaning of your answer in terms of water flow.

(b) Is there some time t, $0 < t < 40$, such that $W'(t) = 0$?

(c) The rate of the water flow $W(t)$ can be approximated by $X(t) = \frac{1}{210}\left(2001 + 39t - t^2\right)$. Use $X(t)$ to approximate the average rate of water flow during the 40-minute time period. Indicate units of measure.

x	1.0	1.2	1.4	1.6	1.8
$f(x)$	5.0	3.5	2.6	2.0	1.5

18. Consider the table above, showing values for the differentiable function f.

(a) Estimate $f'(1.4)$.

(b) Give an equation for the tangent line to the graph of f at $x = 1.4$.

(c) Is $f''(x)$ positive, negative, or zero? Explain how you determine this.

(d) Using the data in the table, find a midpoint approximation with 2 equal subintervals for $\int_{1.0}^{1.8} f(x)\,dx$.

19. A racehorse is running a 10 furlong race (1 furlong is 220 yards). As the horse passes each furlong marker, F, a steward records the time elapsed, t, since the beginning of the race, as shown in the table below.

t	0	20	33	46	59	73	86	100	112	124	135
F	0	1	2	3	4	5	6	7	8	9	10

(a) How long does it take the horse to finish the race?

(b) What is the average speed of the horse over the the first 5 furlongs?

(c) What is the approximate speed of the horse as it passes the 3-furlong marker?

(d) During which portion of the race is the horse running the fastest? Accelerating the fastest?

20. The rate at which water flows out of a pipe is given by a differentiable function R of time t. The table below records the rate at 4-hour intervals for a 24-hour time period.

t (hrs)	0	4	8	12	16	20	24
$R(t)$ (gal/hr)	14.6	15.3	15.9	16.1	15.9	15.5	14.6

(a) Use the Trapezoid rule with 6 subintervals of equal length to approximate $\int_0^{24} R(t)\,dt$. Explain the meaning of your answer in terms of water flow. Indicate units of measure.

(b) Is there some t, $0 \le t \le 24$, such that $R'(t) = 0$? Justify!

(c) Suppose the rate of water flow is approximated by the differentiable function $Q(t) = 0.01\left(1460 + 25x - x^2\right)$. Use $Q(t)$ to approximate the average rate of water flow during the 24-hour period. Indicate units of measure.

21. Let f be an even, twice-differentiable function which is continuous on the interval $-6 \le x \le 6$. The function f and its derivatives have the properties indicated in the table below.

x	0	$0 < x < 2$	2	$2 < x < 4$	4	$4 < x < 6$
$f(x)$	2	positive	0	negative	-2	negative
$f'(x)$	0	negative	-1	negative	0	positive
$f''(x)$	negative	negative	0	positive	positive	positive

(a) For $-6 \le x \le 6$, identify all the values of x at which f attains a local minimum. Justify your answer.

(b) For $-6 \le x \le 6$, identify all the values of x at which f has a point of inflection. Justify your answer.

(c) For $-6 \le x \le 6$, identify the coordinates of any local maximums of f. Justify your answer.

22. The number of people in line at the Department of Driver Services is modeled by a twice-differentiable function $P(t)$ for $0 \le t \le 12$. The time $t = 0$ corresponds to 7 AM. The table below shows certain values of $P(t)$.

t (hours)	0	1	2	5	7	10	12
$P(t)$ (people)	25	27	18	45	25	30	35

(a) Using the data in the table, estimate the rate at which the number of people in line was changing at 10:30 AM ($t = 3.5$). Indicate units of measure and show the computations which lead to your answer.

(b) Using a right-hand Riemann sum with subintervals as indicated by the table, estimate $\frac{1}{12}\int_0^{12} P(t)\,dt$. Explain the meaning of this integral in the context of the problem.

(c) For $0 \le t \le 12$, what is the least number of times that $P'(t) = 0$? Justify your answer.

23. The table below shows some values for a twice-differentiable function g.

x	-1	0	1	2	3	4	5
$g(x)$	8	5	-2	1	-1	-3	3

(a) Approximate $\int_{-1}^{5} g(x)\, dx$ using a left Riemann sum over 3 subintervals of equal length.

(b) Using the aprroximation from part (a), estimate the average value of g over $[-1, 5]$.

(c) Find the average rate of change of g over $[-1, 5]$.

(d) For $-1 < q < 1$, explain why there must exist q such that $g(q) = 0$.

(e) For $-1 < r < 1$, explain why there must exist r such that $g'(r) = -5$.

24. The functions g and h are differentiable for all real numbers. The table below gives values of g and h and their first derivatives. The function f is given by $f(x) = 2g\,(h(x)) - 17$.

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
0	5	1	6	-3
1	8	3	5	-1
2	11	6	-1	1
3	-2	2	-3	4

(a) For $0 < c < 3$, must there be a value of c such that $f(c) = 2$? Justify your answer.

(b) For $0 < d < 3$, must there be a value of d such that $f'(d) = 2$? Justify your answer.

(c) Using values as given in the table, compute $f'(3)$.

25. The function $R(t)$ models the rate of change in the value of a company's stock, in dollars per day, over a 12-day period. Some values of R are shown in the table below.

t (days)	0	2	5	9	11	12
$R(t)$ (\$/day)	10	22	15.77	13.29	12.70	12.48

(a) Using a right Riemann sum with subintervals as indicated in the table, estimate $\int_0^{12} R(t)\, dt$. Explain the meaning of the integral using correct units.

(b) What is the average rate of change in $R(t)$ over the interval $[0, 12]$?

(c) Using the estimate from part (a), what is the average value of $R(t)$ over $[0, 12]$? Indicate units of measure.

26. An object moves in a straight line for 12 seconds. The velocity, in meters per second, and the acceleration, in meters per second per second, are differentiable functions. The table shows selected values of the functions.

t (sec)	0	2	5	6	10	12
$v(t)$ (m/s)	0	4	8	6	10	30
$a(t)$ (m/s^2)	1	3	2	4	1	3

(a) Using the subintervals as indicated in the table, use a trapezoidal sum to estimate $\int_0^{12} v(t)\,dt$.

(b) Explain the meaning of $\frac{1}{12}\int_0^{12} v(t)\,dt$ using correct units.

(c) Approximate $\int_0^{12} a(t)\,dt$ using a trapezoid sum with subintervals as indicated in the table. What is the actual value of $\int_0^{12} a(t)\,dt$?

(d) Explain why acceleration must equal $\frac{1}{2}$ for some value of t in the interval $2 \le t \le 6$.

27. Let f be a continuous function defined on the interval $-2 \le x \le 3$. The function is twice-differentiable, except at $x = 1$. The function and its derivatives have properties as defined by the table below.

x	-2	$-2 < x < -1$	-1	$-1 < x < 1$	1
$f(x)$	-2		-1		1
$f'(x)$	5	positive	0	positive	d.n.e.
$f''(x)$	-2	negative	0	positive	d.n.e.

x		$1 < x < 2$	2	$2 < x < 3$	3
$f(x)$			3		1
$f'(x)$		positive	0	negative	-4
$f''(x)$		negative	0	negative	3

Both $f'(1)$ and $f''(1)$ do not exist.

(a) On $[-2, 3]$, find the x-coordinates of any absolute extrema for the graph of f. Justify your answer.

(b) On $[-2, 3]$, find the x-coordinates of any points of inflection for the graph of f. Justify your answer.

(c) Let $g(x) = x^2 f(x)$. Write the equation of the line tangent to the graph of g at $x = -1$.

(d) Sketch a possible graph of f over the interval $[-2, 3]$, using the properties from parts (a) and (b).

28. The function f is twice-differentiable and properties of f' and f'' are given in the table below.

x	$-2 < x < 1$	1	$1 < x < 4$	4	$4 < x < 5$
$f'(x)$	positive	1	positive	0	negative
$f''(x)$	positive	0	negative	0	negative

Determine whether each statement below must be true or could be false. Explain your reasoning for each statement.

I. The graph of f has a relative maximum at $x = 1$ and an inflection point at $x = 4$.
II. The graph of f has a relative maximum at $x = 4$ and an inflection point at $x = 1$.
III. The graph of f has a relative maximum at $x = 4$ and no inflection point.
IV. The graph of f has a relative minimum at $x = 1$ and an inflection point at $x = 4$.
V. The graph of f has a relative minimum at $x = 1$ and no inflection point.

29. The table below shows four points on the differentiable function $f(t)$.

t	$f(t)$
8	8.253
9	8.372
10	8.459
11	8.616

(a) Estimate $f'(10)$.

(b) Estimate $f^{-1}(8.5)$

(c) Estimate $\int_8^{10} f'(t)\, dt$.

(d) Estimate $\left(f^{-1}\right)'(8.4)$.

30. The table below shows data collected in an experiment of a new type of electric engine for a small vehicle. The velocity measurements, in miles per hour, were taken in 15-minute intervals on a 2-hour trip.

t	0	0.25	0.50	0.75	1.00	1.25	1.50	1.75	2.00
$v(t)$	0	10.30	13.10	13.50	16.20	19.10	20.20	15.30	9.60

(a) What is the average acceleration over the interval $[0.25, 0.75]$? Indicate units of measure.

(b) Explain the meaning of $\frac{1}{2}\int_0^2 v(t)\, dt$ using correct units.

(c) Approximate the integral in part (b) using a midpoint Riemann sum with four equal subintervals.

(d) The velocity of the vehicle on the interval $0 \le t \le 2$ can be modeled by the function $V(t) = -13t^4 + 52t^3 - 74t^2 + 50t$. Use this model to determine the acceleration of the vehicle at $t = 0.5$. How does this compare to your answer in part (a)? Explain.

Thanks!

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