Serving STEM Students After AP Calculus

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May 15, 2018

A common issue facing STEM schools is what to do with students who take AP Calculus as juniors (or earlier). Schools and school systems have various ways to support these students. Some schools simply offer a year of AP Calculus AB followed by a year of AP Calculus BC; some offer AP Statistics after (or before) calculus; and some schools have the opportunity to be located near a college or university which offers dual enrollment. For various reasons, none of these options worked for the Rockdale Magnet School for Science and Technology, and it was left to us to determine how to serve these students.

A Little History

To understand why these typical options did not work for us, a little background may be helpful. When RMSST was founded in 2000, the only calculus class offered was AP Calculus BC, and it was required to graduate. By the 2002-03 school year, we had juniors in BC. As seniors, they went to a local campus of a large community college to take Discrete Mathematics. We did this for only two years because the college discontinued offering Discrete Mathematics at that campus. The RMSST Advisory Board requested that a course be offered in the school for these students. Around the same time, the school grew larger and we also decided to offer a math elective. Two senior-level, year-long courses grew out of this initiative: Multivariable Calculus as the course required of seniors who took BC as juniors, and the History of Mathematics as an elective course.

Neither of these courses existed, so it was up to me to write the curriculum and shepherd them through the State Board of Education approval process. The courses were approved and were made available to any school in Georgia as official state courses. The first course to win approval was History of Mathematics, as it was easier to gain approval for a pure elective than a “required” math class. It was first offered at our school in the 2003-04 school year, and has been offered every year since. Multivariable Calculus took a little longer to be approved and was first offered in 2005-06. Soon after, our administration at the time wanted

to give students a little more variety in their courses selections. I recalled the Discrete Math course taken by our seniors and decided to cut History of Math down to a one-semester course, paired with a semester of Discrete Math. This proved to be a wise decision, as this pairing proved popular enough with seniors to hit maximum enrollment each year. We offered this pairing from 2006-07 until 2010-11. The reason we stopped was that the Discrete Math course, an approved state course for decades, was removed from the course list, due to the gradual introduction of the Georgia's new standards-based curriculum, called the "Georgia Performance Standards." (Now the standards are called "Georgia Standards of Excellence".) This change in curriculum affected History of Math and Multivariable Calculus as well, as they each had to be re-written to be standards-based. I rewrote and submitted them to the state Department of Education for approval again. They were happily approved again as well. (Full disclosure: at this time I served on the Department of Education's Math Advisory Council, which approves all mathematics initiatives and courses. I recused myself when these courses came up for discussion and vote. I continue to serve on the Advisory Council.)

However, Discrete Math was simply removed, and I heard that no such course would ever be approved due to ever-decreasing rigor in non-STEM schools. Seeking a new course to pair with the semester History of Math, I learned of a new course offered by the state, called Mathematics of Industry and Government (MIG). This course was really an introduction to operations research using Microsoft Excel. This seemed like a good fit with our school's new one-to-one device initiative. We started offering MIG with History of Math in 2011-2012, and have offered these electives every year since.

The latest change is the course called "Advanced Finite Mathematics" (AFM). This course is really a discrete mathematics course by another name. I heard from alumni who told me that the old discrete math was "the most useful math class" they took. Their justifications for this bold statement fell in two categories: that learning to write proofs helped them with all other math classes they encountered, and that it aroused their interest in other branches of mathematics besides calculus. Based on their experience, as well as wanting to offer a course for students who did not want another year of calculus, I designed and wrote the curriculum for the year-long AFM course and it was approved by the state Department of Education. We offered the course for the first time in the 2016-17 school year.

All the courses that are described are approved, state-funded courses for any high school in Georgia to make available to their students.

Multivariable Calculus

This is possibly the most traditional option for seniors who took AP Calculus as juniors. However, the curriculum for this course includes an introduction to linear algebra and to differential equations. The following is a summary of the standards.

- Students will investigate the relationship between points, lines, and planes in three-dimensions.
- Students will recognize and apply properties of matrices. Students will explore functions of two independent variables of the form $z = f(x, y)$ and implicit functions of the form $f(x, y, z) = 0$.
- Students will explore the continuity of functions of two independent variables in terms of the limits of such functions as (x, y) approaches a given point in the plane.
- Students will explore, find, use, and apply partial differentiation of functions of two independent variables of the form $z = f(x, y)$ and implicit functions of the form $f(x, y, z) = 0$.
- Students will define and apply the gradient, the divergence, and curl in terms of differential vector operations.
- Students will integrate functions of the form $z = f(x, y)$ or $w = f(x, y, z)$.
- Students will apply and interpret the theorems of Green, Stokes, and Gauss.
- Students will use, apply, and solve linear first-order and second-order differential equations.

Clearly, there is much that students must learn in order to "apply and interpret the theorems of Green, Stokes, and Gauss." Students must learn include line integrals in the plane, line integrals in space, surface integrals, vector and parametric representations of functions, and the notion of independence of path. There is a lot of knowledge packed into these few standards.

History of Mathematics

This course has been one of the more popular electives at our school. Alumni often say that it is an interesting class, and they say the class has broadened their notion of mathematics itself and its impact on our society. From the outset, I wanted the course to be more than just a history of math up to calculus, I wanted to go up to the present. That means teaching students some challenging mathematical topics—such as non-Euclidean geometry and group theory—so they may gain a better perspective on the historical impact of these topics. With only a semester to span all of human history, some things must be left out. So my perspective in teaching the course is from the student's: why do we do the math we do, and how did it get this way? This simple question guides what I teach and how. The following are the broad standard topics.

- Students will explore and use historical methods for expressing and solving equations.

- Students will explore abstract algebra and group-theoretic concepts.
- Students will use and apply number theoretic concepts.
- Students will use the algebraic techniques of Fermat, Barrow, and Newton to determine tangents to quadratic curves.
- Students will prove geometry theorems.
- Students will compute lengths, areas, and volumes according to historical formulas.
- Students will explore and prove statements in non-Euclidean geometry.
- Students will compute the ratio of winnings in an interrupted game.
- Students will identify Hindu-Arabic numerals as a prime scientific advancement.
- Students will describe factors involved in the rise and fall of ancient Greek society.
- Students will trace the centers of development of mathematical ideas from the 5th century to the 18th century.
- Students will identify the 19th and 20th centuries as the time when mathematics became more specialized and more rigorous

Mathematics of Industry and Government

This is the one course in this article I did not write. The course is an introduction to operations research through the use of Excel. Students are given problems in linear, integer, and binary programming to solve using Excel's Solver add-in. Students are taught how manage Excel formulas and sheets to set up the problem so that Excel can solve it and so that the solutions can be read by humans! The course is actually a year-long course, but we only offer the first semester, which is optimal programming and deterministic decision making. The standards are below. The second semester is probabilistic decision making.

- Students will use advanced linear programming to make decisions.
- Students will determine optimal locations and use them to make appropriate decisions.
- Students will determine optimal paths and use them to make appropriate decisions.

Advanced Finite Mathematics

This course introduces a variety of discrete mathematical structures, and interweaves proofs throughout. Students initially took this course instead of Multivariable Calculus, because they thought it would be easier, but writing good proofs presents its own unique challenges. This course includes so much that to list it all would take pages. Listed below are not the standards, but the overall group heading of the standards, so you have some idea of the breadth of the course.

- Represent and interpret statements using logical symbolism
- Use set theoretic operations
- Use and interpret Boolean algebra
- Use number theoretic operations
- Prove statements in number theory
- Apply number theory
- Calculate the probability of events
- Use methods of counting
- Prove statements involving combinatorics
- Use and recognize graph properties
- Prove statements in graph theory
- Apply graph theory
- Use certain proof techniques

For example, under “proof techniques” one finds explicit standards for teaching mathematical induction (both regular and strong induction), proof by contradiction, and proof by contrapositive. There is quite a lot involved in AFM.

Conclusion

We have mathematics courses at our school which make us unique. The high standards of rigor, while difficult to maintain consistently, give our students a unique advantage: some alumni have successfully argued that they should be exempt from certain college courses (one successfully argued for credit). The obvious benefit to creating courses yourself is that you know your student populations and you can design a course to serve that population well. But there is another benefit: professional development. To write a course, you really have to know the material, know the most important parts and themes, and communicate that to

students and colleagues. One must interact with other teachers and educators, to get feedback and advice. In doing so, I made contacts at the Department of Education which led to opportunities to present at state and national conferences, and to sit on state and national Boards. This is a rewarding experience which I would suggest anyone do if given the opportunity.

For more detail into the all of these standards of these four courses, see https://www.georgiastandards.org/Georgia-Standards/Pages/Math-9-12.aspx. For more information as to how I teach the course, see my webpages, http://www.drchuckgarner.com.