

# The Thinker and the Thug

## The History of Mathematics, Part 9

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# Outline

## The Life of Archimedes

## The Works of Archimedes

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On Conoids and Spheroids

On Spirals

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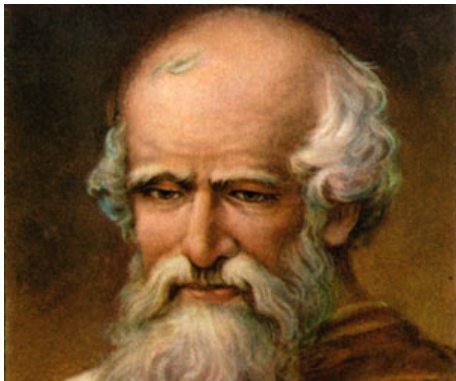
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# Archimedes



*Archimedes*

287 BC-212 BC

*“Give me a place to stand and I will move the Earth.”*

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# Archimedes of Syracuse

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# Archimedes

- ▶ Spent youth in Alexandria
- ▶ Related to King Hieron; advisor to the King
- ▶ Inventor and mathematician
- ▶ Unique because he combined pure geometrical analysis and the mechanical/practical (Plato urged separation of the two)

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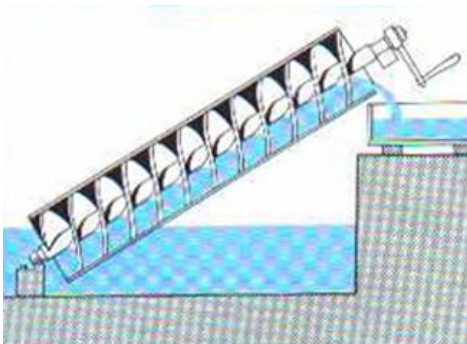
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The Archimedean Screw

# Archimedes' Inventions

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The Archimedean Screw in use at a water treatment plant in Tennessee



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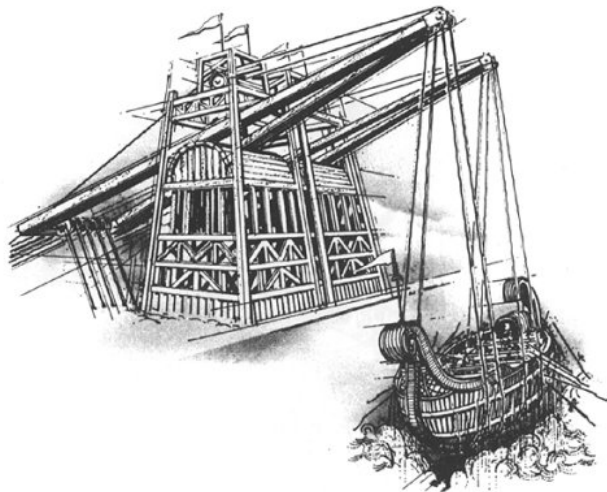
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The Archimedean Claw

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- ▶ Discovered the first law of hydrostatics:  
“A body immersed in fluid is buoyed up by a force equal to the weight of the displaced fluid.”
- ▶ Invented the pulley
- ▶ Analyzed the lever
- ▶ Died during the Roman siege of Syracuse

# Archimedes

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## The Life of Archimedes

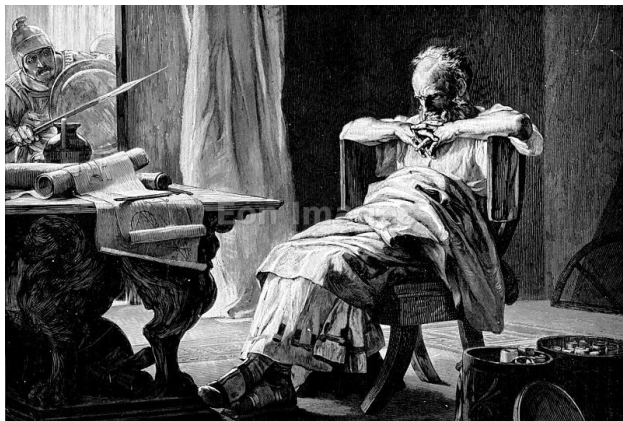
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*"Do not disturb my circles!"*

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# The Surviving Works of Archimedes

- ▶ *On the Sphere and Cylinder* (2 books)
- ▶ *Measurement of a Circle*
- ▶ *On Conoids and Spheroids*
- ▶ *On Spirals*
- ▶ *On the Equilibrium of Planes* (2 books)
- ▶ *The Sand-Reckoner*
- ▶ *Quadrature of the Parabola*
- ▶ *On Floating Bodies* (2 books)
- ▶ *Book of Lemmas*
- ▶ *The Cattle-Problem*

# The Surviving Works of Archimedes

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- ▶ All works come to us with cover letters:
  - ▶ Many to Dositheus
  - ▶ One to Eratosthenes, librarian at Alexandria
- ▶ We get insight into mathematical atmosphere of the time
- ▶ We get insight into Archimedes himself

# On the Sphere and Cylinder

- ▶ Archimedes own favorite work
- ▶ 2 books, 60 propositions

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# On the Sphere and Cylinder

- ▶ Archimedes own favorite work
- ▶ 2 books, 60 propositions
- ▶ Propositions 33 and 34 of Book I:

The cylinder whose base is equal to a great circle of the sphere and whose altitude is equal to a diameter of the sphere has a total surface exactly equal to  $\frac{3}{2}$  the surface of the sphere and a volume exactly equal to  $\frac{3}{2}$  the volume of the sphere.



# On Conoids and Spheroids

- ▶ Calculated volumes and areas of paraboloids, cones, hyperboloids, spheroids
- ▶ Proved that the volume of a paraboloid is  $\frac{3}{2}$  the volume of a cone with the same base and axis

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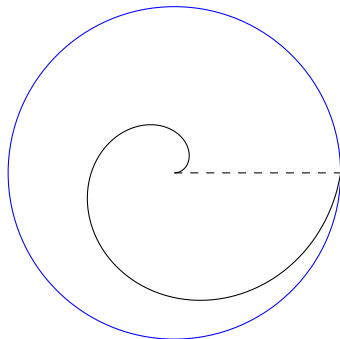
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- ▶ Area of one circuit is one-third the area of the circle with radius equal to the distance of the endpoints

# On the Equilibrium of Planes

Represents the first instance of mathematical modeling.

**Post. 1** Equal weights at equal distances are in equilibrium; equal weights at unequal distances incline toward the weight at the greater distance. (Principle of Insufficient Reason)

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# On the Equilibrium of Planes

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**Post. 1** Equal weights at equal distances are in equilibrium; equal weights at unequal distances incline toward the weight at the greater distance. (Principle of Insufficient Reason)

**Post. 2, 3** When weights at certain distances are in equilibrium, and something is (added to/subtracted from) one of the weights, they will incline toward the weight (to which the addition was made/from which nothing was taken).

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**Prop. 1** Weights which balance at equal distances are equal.

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**Prop. 1** Weights which balance at equal distances are equal.

**Prop. 3** Suppose  $A$  and  $B$  are unequal weights with  $A > B$  which balance at  $C$ . Then  $AC < BC$ . Conversely, if the weights balance and  $AC < BC$ , then  $A > B$ .

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**Note** The lever is not mentioned, it is just “there.”

# The Sand-Reckoner

- ▶ Recall: myriad represents 10,000
- ▶ Proposed a number system using powers of myriad myriads (base 100,000,000)
- ▶ Calculates  $8 \times 10^{63}$  as the number of grains of sand to fill the universe

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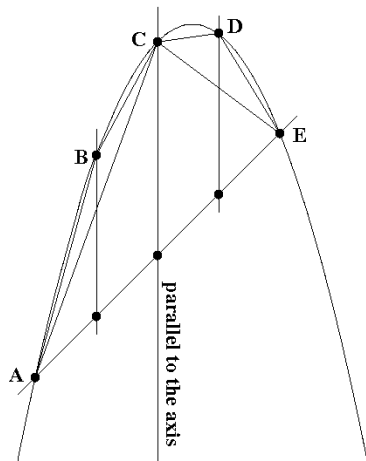
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# Quadrature of the Parabola



The area of a parabolic segment ( $ABCDE$ ) is  $\frac{4}{3}$  the area of the inscribed triangle ( $\triangle ACE$ ).

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# Other Works

**On Floating Bodies** Hydrostatics and buoyancy

**Measurement of a Circle** Used circumscribed and inscribed regular polygons; with 96-gons, obtained  $3\frac{10}{71} < \pi < 3\frac{1}{7}$

**Book of Lemmas** Geometry; **15 Propositions** ([link](#))

**On Sphere-Making** now lost, on constructing a planetarium

**The Cattle-Problem** Number theory

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# The Cattle-Problem

$W, w = \textit{white bulls, cows}$

$X, x = \textit{black}$

$Y, y = \textit{yellow}$

$Z, z = \textit{spotted}$

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# The Cattle-Problem

$W, w =$  *white bulls, cows*

$X, x =$  *black*

$Y, y =$  *yellow*

$Z, z =$  *spotted*

subject to

$$W = \frac{5}{6}X + Y$$

$$Z = \frac{13}{42}W + Y$$

$$x = \frac{9}{20}(Z + z)$$

$$y = \frac{13}{42}(W + w)$$

$$X = \frac{9}{20}Z + Y$$

$$w = \frac{7}{12}(X + x)$$

$$z = \frac{11}{30}(Y + y)$$

# The Cattle-Problem

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$Z, z =$  spotted

subject to

$$W = \frac{5}{6}X + Y$$

$$X = \frac{9}{20}Z + Y$$

$$Z = \frac{13}{42}W + Y$$

$$w = \frac{7}{12}(X + x)$$

$$x = \frac{9}{20}(Z + z)$$

$$z = \frac{11}{30}(Y + y)$$

$$y = \frac{13}{42}(W + w)$$

$W + X$  is a square and  $Y + Z$  is a triangle

# The Cattle-Problem

For the first seven conditions:

$$W = 10366482n$$

$$X = 7460514n$$

$$Y = 4149387n$$

$$Z = 7358060n$$

$$w = 7206360n$$

$$x = 893246n$$

$$y = 5439213n$$

$$z = 3515820n$$

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To be square,  $n = 4456749k^2$ .

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To be square,  $n = 4456749k^2$ .

Finally,  $W$  alone has 206541 digits!

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Discovered in 1906 in Constantinople

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Prayer book assembled in 1229 from Greek copy of  
Archimedes' works

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This palimpsest contains many of Archimedes' works already known, but also includes

- ▶ Nearly all of *On Floating Bodies*, most of which was lost
- ▶ Most of a book called “*Ἐπιπέδος*”, meaning *Method*

# The Method

- ▶ Archimedes tells how he came to discover his theorems
- ▶ Careful to make the distinction between “investigation” and “proof”
- ▶ Developed volume of sphere using a technique from Eudoxus: the method of exhaustion
- ▶ **The Archimedes Palimpsest (link)**

# Two Aspects of Numbers

1. Relationship among numbers — “Arithmetic”  
(Today we call this *number theory*)
2. Computing with numbers — “Logistic”  
(Today we call this *arithmetic*)

# Two Aspects of Numbers

1. Relationship among numbers — “Arithmetic”  
(Today we call this *number theory*)
2. Computing with numbers — “Logistic”  
(Today we call this *arithmetic*)

Greek tendency to think of aspects in pairs:

- ▶ Even/Odd
- ▶ Measure/Number
- ▶ Magnitude/Multitude
- ▶ Arithmetic/Logistic
- ▶ Commensurable/Incommensurable

Archimedes brought these aspects together.

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# Archimedes' Legacy

- ▶ The greatest genius of antiquity
- ▶ One of the 3 greatest mathematicians ever
- ▶ Laid foundation for integral calculus
- ▶ Style influenced all natural philosophers for millenia

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# Archimedes' Legacy



Statue on eastern coast of Syracuse, Sicily, erected in  
2016

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# Homework

- ▶ Last-Minute Problems, #4 – due March 1
- ▶ Chasing the value of  $\pi$ ;  
*Math Through the Ages*, Sketch 7

Next: A Look Toward the Heavens

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