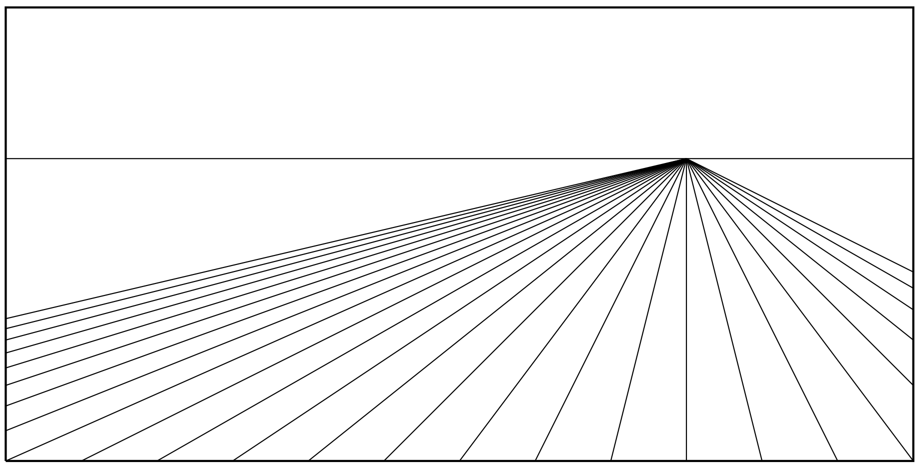
Last-Minute Problems, No. 7

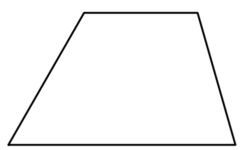
1 TEXTBOOK PROBLEMS, PART ONE. [4] p.101 #1 and #2.

2 TEXTBOOK PROBLEMS, PART TWO. [15] p.207 #1, #2, #3, #4, and #6.

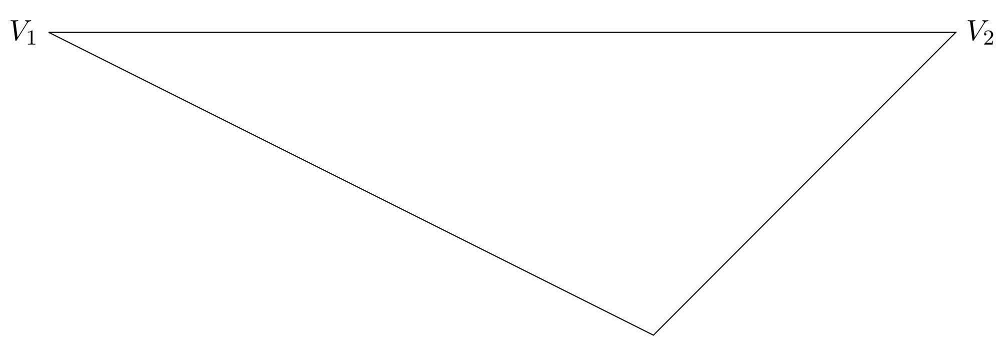
3 KEEPING THINGS IN PERSPECTIVE. [5] Either on this paper or by tracing the figure, construct at least five lines parallel to the horizon line such that you create a perspective view of a tiled floor.



4 A PERSPECTIVE PUZZLE. [5] Suppose the diagram below represents a square tile in perspective view. Explain how to find the correct perspective view of the midpoints of the sides and the center of the square.

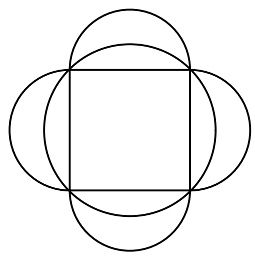


5 KEEPING THINGS IN PERSPECTIVE AGAIN. [5] Either on this paper or by tracing the figure, construct at least three lines from each vanishing point (V_1 and V_2) such that you create a perspective view of a tiled floor.



6 Da Vinci's "Dynamic Dissection of the Lune". [8] Leonardo Da Vinci (1452-1519) is most famously known as an artist, but he was an intellectual who sought any and all knowledge. He was an inventor, an artist, a philosopher, and a technological innovator. He is widely recognized as one of the most diversely talented individuals ever to have lived. However, it still comes as a surprise to many that he was also interested in mathematics. Here is an algebraic version of Da Vinci's proof of the quadrature of the lune.

6a) Begin with a square of side length $2r$ and upon each side contruct a semicircle. Simultaneously circumscribe a circle about the square, as shown below. Find the combined area of the four semicircles in terms of r.



6b) Now find the area of the circumscribed circle in terms of r. Notice anything?

6c) According to parts 6a and 6b, we get the same area if we remove the circumscribed circle from the figure above as we get if we instead remove the four semicircles from the figure above. Use this to explain why a single lune from the picture is quadrable.

7 The Regular Pentagon. [8] While you have your straightedge out for some of the other problems, get a compass as well. In this problem, you will construct a regular pentagon. If the regular pentagon is inscribed in a circle of radius 1, the side length of the pentagon is $\sqrt{10 - 2\sqrt{5}}/2$; this is thought to be another one of the early incommensurable magnitudes discovered by the Pythagoreans.

7a) Begin by constructing a circle. How big doesn't really matter; just make it easy to draw with your compass. Label the center of your circle C.

7b) Construct a diameter; label the endpoints A and B.

7c) Construct radius $\overline{CF}$ perpendicular to $\overline{AB}$.

7d) Bisect $\overline{BC}$ at D.

7e) With center D and radius $\overline{DF}$ draw an arc cutting $\overline{AB}$ at E. Construct $\overline{EF}$.

7f) With center F and radius $\overline{EF}$ draw an arc cutting the circle in two places, at G and J. Construct segments $\overline{FG}$ and $\overline{FJ}$.

7g) The segments $\overline{FG}$ and $\overline{FJ}$ are two adjacent sides of the regular pentagon. Finish constructing the other sides of the pentagon by drawing arcs: one with center G and radius $\overline{FG}$ cutting the circle at H, and another with center J and radius $\overline{FJ}$ cutting the circle at I. Then $FGHIJ$ is a regular pentagon.

7h) *Bonus for the Daring:* Why does this construction work?