Five Weeks to a Five

Preparation for the AP Calculus AB Exam

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RMMT Publishing 2019

Five Weeks to a Five: Preparation for the AP Calculus AB Exam

Published by RMMT Publishing, Conyers, Georgia

First edition, 2019

Set in *STIX*, the Scientific and Technical Information Exchange Font (based on Times New Roman), designed by the STIPub Consortium in 2013, and Clear Sans, designed by Daniel Ratighan in 2013.

This book was produced directly from the author's LaTeX files.
Figures were drawn by the author using the TeXdraw and PSTricks packages.
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Preface

This book is designed to prepare a student for taking the AP Calculus AB Exam.

Another exam prep book? There are so many!

Well, yes, there are quite a few available. But they are written for two purposes. Either they are simply collections of practice exams and advice, or they are for self-study. None of them seem to be written with a classroom teacher and their students in mind.

So you're saying this one is different, huh? How so?

I assume that the teacher using this book has spent the school year teaching calculus to their students, and now it is time to prepare for the exam. Through this book's 12 Lessons, students can focus on one aspect of the AP Calculus curriculum from the viewpoint of how it will be assessed on the AP Exam. This is important: the material is presented in terms of how it will be assessed. For example, a calculus teacher probably spent a few class periods teaching students how to use the information from the first and second derivatives to sketch the graph of a function. This should be taught and is important—however, it has been nearly two decades since any free-response question has asked students to sketch the graph of a function. This is simply no longer assessed. (Other graphical aspects of using the derivative are definitely assessed, but not curve sketching.) So curve-sketching is not included in the Lessons, in the problems, or in the practice exams. This is unlike those other books.

What do you mean unlike those other books? Are they filled with curve-sketching problems?

Well, no, but they are filled with interesting things. I have used many different AP Exam Prep books over my career teaching calculus. Two popular books of practice exams in particular have been my "go-to" resource. Until I actually looked at the problems and compared them to the released AP exams. They are not similar in style, and, in some cases, content. These books include curve-sketching free-response problems, antiquated related rates problems, free-response problems where the parts of the problem are too trivial to appear on the AP Exam, and, in one of them, there is even a free-response problem which is all precalculus, except for one part. And yet, these books are supposed to prepare students for the AP Exam. (I'm not saying these problems are not important, or they do not provide good practice, or there is not a pedagogical rationale for asking students to solve them—but questions which do not reflect what students can expect to see on the actual exam isn't really what I'd call "preparation".)

And this doesn't include the other interesting things I find in these books. I went to a Barnes & Noble looking for other possible AP Calculus prep books when I realized my current "go-to"s were not really adequate. I was stunned to see problems on topics no longer in the AP Calculus AB Course Description, such as work problems, marginal cost, and linear first-order differential equations. I saw many multiple-choice questions with directions such as "Use the graph below to answer problems 25-28" which would *never* be found on the AP exam. I also saw BC questions in an AB practice exam! And people buy these books by the thousands.

Then I realized that for a book which should prepare you for the exam, none of them actually *look like the AP Exam!* For instance, the multiple-choice questions are not spaced apart with room to work and separated by horizontal rules, and in those that do space the problems apart, there is an "answer box" for students to write their choice (which is *not* how the AP Exam works). The free-response questions are not printed within a border with the parts separated by a horizontal rule. The graphs and figures are not professionally designed, but instead are copy-and-paste jobs from whatever software the publishers happen to be using! It isn't hard to use the correct typeface, suitable figures, and appropriate spacing to mimic the layout of the AP Exam! If we are going to prepare students for the AP Exam, then let's really get them prepared by offering something that looks like the exam!!

Wow—You sound a little frustrated. It will be okay!

Sorry. But I get so worked up over the lack of thought and care that poor typesetting and formatting indicates!

Allrighty then... So what else is so unique about this book? (Besides this insane Preface which appears to be a dialogue between the personalities in the author's head.) How many problems are there?

At the end of each of the 12 Lessons are 15 multiple-choice problems and 2 free-response problems. There are also four complete practice exams, each with 45 multiple-choice questions and 6 free-response questions. Thus, there are $12 \cdot 15 + 4 \cdot 45 = 360$ multiple-choice questions and $12 \cdot 2 + 4 \cdot 6 = 48$ free-response questions.

That's a lot of problems! Are there answers in the back...?

No. But there is a solution book available to anyone for an outrageously expensive price. The same solution book is available to teachers who bought at least 10 copies of this book for a ridiculously low price.

How do I get the ridiculously low price?

Let me tell you! Teachers emailing me their Lulu.com shipping confirmation which indicates the purchase of at least 10 copies of this book will be emailed a link to purchase the solutions book at the ridiculously low price (plus shipping). This low price is not searchable and can only be accessed by the link I send. The teacher must email the shipping confirmation (that is *shipping confirmation* not receipt of payment!) from their school's email account and it must be sent to `cgarner@gctm.org`. If the teacher does not feel comfortable sending their personal information as listed on the shipping confirmation, they are free to black out or remove that information, as long as the order number, quantity, title, order date, and Lulu.com logo are visible.

Oh, ok—thanks. So how can I use this book with a class?

Each Lesson is designed to be presented/covered/taught/reviewed in one class period, with homework or classwork assinged from the Lesson. As I teach on a block schedule with 90-minute classes meeting every other day, I assign each Lesson as classwork and portions of 3 of the 4 practice exams as homework. I use the fourth practice exam as classwork. This takes me 14 block days, and since we meet every other day, this takes five weeks.

So that's why the title is *Five Weeks to a Five*! I was wondering what's up with that.

Yes, well, it's the best title I could think of that used "five" which wasn't already taken. Meh.

Anyway, the reason each Lesson has 15 multiple-choice and 2 free-reponse is that by assigning three Lessons, students have done the equivalent of an AP Exam ($3 \cdot 15 = 45$ multiple-choice and $2 \cdot 3 = 6$ free-response). Indeed, 4 of the 12 Lessons are allowed the use of a calculator: The Fundamental Theorem of Calculus (Lesson 4), Areas (Lesson 5), Motion (Lesson 8), and Volumes (Lesson 11). So really, one could mix-and-match the Lessons to create an AP Exam over only certain topics. Indeed, all one needs to do is choose any two of the non-calculator Lessons and any one of the calculator Lessons, and then one has an appropriate-length exam simulation in the proper calculator-to-non-calculator ratios.

To make an AP Exam-length assignment in the proper calculator-to-non-calculator ratio...

assign the problems in any two of these Lessons...	*and any one of these Lessons.*
Limits and Continuity (Lesson 1)	
Derivatives (Lesson 2)	
Antiderivatives (Lesson 3)	The Fundamental Theorem of Calculus (Lesson 4)
Graphs of Functions (Lesson 6)	Areas (Lesson 5)
Riemann Sums and Trapezoids (Lesson 7)	Motion (Lesson 8)
Other Major Theorems of Calculus (Lesson 9)	Volumes (Lesson 11)
Applications of Differentiation (Lesson 10)	
Differential Equations and Slope Fields (Lesson 12)	

For instance, if I want my students to really focus on integration and its meaning, I would choose to put together Lessons 3, 5, and 7, on Antiderivatives, Riemann Sums, and Areas. That's just one example of the $\binom{8}{2}\binom{4}{1} = 28 \cdot 4 = 112$ possible combinations of exam-length assignments or assessments you could make.

The Lessons and the four preactice exams give you a quantity of problems equivalent to *eight* AP Exams.

Showing off your counting skills there, I see?
You've been going on and on about the problems. What about the Lessons themselves? If this book isn't for self-study, and a teacher is already teaching the class with a textbook, then why have "lessons" at all?

The Lessons serve as another viewpoint for the students (and possibly the teacher). I have tried to explain topics and theorems solely from the viewpoint of how they will be assessed. This is not a hard-and-fast rule, but more of a guideline. I've also tried to be concise and get straight to the point, without all the extra fluff; that can be expanded upon by the teacher if need be. Finally, the examples are completey worked out, and include some tips on taking the exam. Oh! I've also tried not to rely on any one kind of calculator by staying away from syntax and by not telling you which buttons to press—the teacher can help you there. Besides, I don't know which kind of calculator you're using!

Well, I don't have anymore questions—Wait! I noticed that after all that talk about how a preparation book should "look like" the AP Exam, the problems at the end of each Lesson are not formatted like the exam at all! You've only formatted the practice exams to be like the AP exam! Hypocrite much?

Now hang on—I thought about formatting *all* the problems like the AP Exam, but that would have added another 50 pages and increased the cost. So I was trying to think about the teachers who would be buying this, trying to keep costs down. So this was a conscious decision *not* to format the Lesson problems like the AP Exam.

Hmmm...I'll let it pass this time.

Thanks. Speaking of thanks, let me take this opportunity to thank my students: without them, my life would be boring indeed. And I'd like to thank the Georgia Association of AP Mathematics Teachers (`www.gaapmt.org`): they are a great bunch of friendly and knowledgable teachers who have also taught me a great deal about teaching calculus. And I'd like to thank my wife, Julie, for basically everything.

That was nice! Ok, I'm ready to do some calculus!

Good! Let's get started!

Chuck Garner
CONYERS, GEORGIA
JANUARY 2019

Lesson 9

Other Major Theorems of Calculus

There are *seven* major theorems of calculus. We have already encountered some of these major theorems of calculus: l'Hôspital's Rule (Lesson 1), Differentiability Implies Continuity (Lesson 2), the Fundamental Theorem of Calculus (Lesson 4), the Average Value Theorem (Lesson 4), and the Extreme Value Theorem (Lesson 6). In this Lesson, we take a look at the last two of the seven: the Intermediate Value Theorem and the Mean Value Theorem. We begin with the statement of the Intermediate Value Theorem.

THEOREM 9.1: Intermediate Value Theorem

Let $f(x)$ be a continuous function on the interval $[a, b]$, where $f(a) \neq f(b)$. Then for every k in $(f(a), f(b))$ there is a point c in $[a, b]$ such that $f(c) = k$.

The Intermediate Value Theorem (which we shall abbreviate as IVT) is our justification for saying that a function must be equal to a certain value on an interval. For instance, suppose we know that the function $g(x)$ is continuous on the interval $[-1, 8]$ and that $g(-1) = 5$, and that $g(8) = 26$. Then there must be an x-value, where $-1 < x < 8$, such that $g(x)$ is equal to any particular y-value between 5 and 26. It is this reasoning which allows us to make the conclusion in the following problem.

Example 9.1: Using the IVT on a Function

Prove that $x - \cos(x) = 0$ for some x in the interval $[0, \frac{\pi}{2}]$.

Solution. Let $g(x) = x - \cos(x)$. Since g is the difference of two continuous functions, g is continuous. Also, $g(0) = 0 - \cos(0) = -1$ and $g(\frac{\pi}{2}) = \frac{\pi}{2} - \cos(\frac{\pi}{2}) = \frac{\pi}{2}$. This implies that the graph of g is below the x-axis when $x = 0$ (in particular, the graph passes through the point $(0, -1)$) and the graph is above the x-axis when $x = \frac{\pi}{2}$ (in particular, it passes through $(\frac{\pi}{2}, \frac{\pi}{2})$). Since g is continuous, the graph must pass through the x-axis somewhere between the x-values 0 and $\frac{\pi}{2}$. Therefore, by the IVT, we can conclude that there is some x in $[0, \frac{\pi}{2}]$ such that $x - \cos(x) = 0$. ∎

Example 9.2: Using the IVT in Practice

The rate at which water flows into a tank is given by $w(t) = \frac{1}{10}\sin(\pi t) + 2$ and the rate at which water flows out of the tank is given by $d(t) = e^{t/10}$. Is there a time t, $0 \le t \le 10$, such that the amount of water in the tank is not changing? Justify your answer.

Solution. Since w is the rate at which water flows in and d is the rate at which water flows out, define $V(t) = w(t) - d(t)$, which gives the rate at which the volume of water in the tank is changing. The amount of water in the tank is not changing if the rate is zero; that is, if $V(t) = 0$ for some t, $0 \le t \le 10$. Thus, $w(t) - d(t) = 0$, or $w(t) = d(t)$. Is there such a t? Yes. Here's why.

Since $V(0) = w(0) - d(0) = 2 - 1 = 1 > 0$ and $V(10) = 2 - e < 0$, we see that on the interval $0 \le t \le 10$ there must be a value of t such that $V(t) = 0$ by the IVT. Hence, there is some t, $0 \le t \le 10$, such that $w(t) = d(t)$. ∎

The next theorem is the last of "The Big Seven Theorems."

THEOREM 9.2: The Mean Value Theorem

Let f be a function continuous on $[a, b]$ and differentiable on (a, b), where $a \ne b$. Then, for some c in (a, b),

$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

Ths theorem says that the average rate of the change of a function f over an interval must be equal to the instantaneous rate of change of f somewhere in the interval. That is, the instantaneous must equal the average somewhere on any differentiable function. There are many ways problems on the Mean Value Theorem (MVT for short) can be phrased.

So here's a bunch of them!

Example 9.3: Using the MVT Directly

The tangent line to the function $f(x) = x + 1/x$ at the point $x = a$, for $1 < a < 5$, is parallel the secant line through the function at $x = 1$ and $x = 5$. Find the value of a.

Solution. Parallel lines have equal slopes. The slope of the tangent is the instantaneous rate of change of f, and the slope of the secant is the average rate of change, and we wish to find the point where these are equal. Since f is continuous and differentiable on the interval, we use the MVT. We have $f'(x) = 1 - 1/x^2$ and

$$\frac{f(5) - f(1)}{5 - 1} = \frac{5 + 1/5 - 1 - 1}{4} = \frac{16/5}{4} = \frac{4}{5}.$$

Using the MVT, $f'(x)$ must equal $\frac{4}{5}$. Hence,

$$\begin{aligned} f'(x) = 1 - \frac{1}{x^2} &= \frac{4}{5} \\ \frac{x^2 - 1}{x^2} &= \frac{4}{5} \\ 5x^2 - 5 &= 4x^2 \\ x^2 &= 5 \\ x &= \pm\sqrt{5}. \end{aligned}$$

However, the interval is $1 < x < 5$, so we reject the negative solution. The value of a is $\sqrt{5}$. ∎

Example 9.4: Using the MVT Carefully

Let $f(x) = x^{2/3}$. Is there a value of c, for $-8 < c < 27$, such that $f'(c) = 1/7$? Justify your answer.

Solution. The function $f(x) = x^{2/3}$ is not differentiable on the interval $[-8, 27]$: the derivative does not exist at $x = 0$. Therefore, the MVT does not apply, and so we do not have any guarantee that the derivative is equal to $1/7$. ∎

Beware! If the hypothesis of the MVT is not checked, we could find an erroneous solution. In the previous example, had we not checked the differentiability of the function, and just plunged directly into the conclusion of the MVT, we would have obtained a supposedly viable solution. Note that the average rate of change of $f(x) = x^{2/3}$ on $[-8, 27]$ can be computed to be

$$\frac{f(27) - f(-8)}{27 + 8} = \frac{9 - 4}{35} = \frac{5}{35} = \frac{1}{7}.$$

Using the MVT without justification then leads to the equation

$$f'(x) = \frac{2}{3}x^{-1/3} = \frac{1}{7}$$

which has the solution $x = (\frac{14}{3})^3 = \frac{2744}{27} \approx 101.63$. The problem is that this solution is not in the interval $[-8, 27]$. So although there is a value of c such that $f'(c) = 1/7$, it is not in the interval, and this solution is not valid. The moral of the story:

You must always check the hypothesis of any theorem you intend to use!

Example 9.5: Using the MVT with a Table

The table below shows values of the differentiable function $f(x)$ for certain values of x.

x	0	2	5	7	10
$f(x)$	-3	4	12	8	15

Is there a value c, for $0 < c < 10$, such that $f'(c) = -2$? Justify your answer.

Solution. Since f is differentiable, it is continuous. Thus, we may use the MVT. If we can find an average rate of change equal to -2, then we may conclude there is such a value. The average rate of change from $x = 5$ to $x = 7$ is equal to -2:

$$\frac{f(7) - f(5)}{7 - 5} = \frac{8 - 12}{2} = -\frac{4}{2} = -2.$$

Hence, we may conclude by the MVT that there is a value of c on the interval $[5, 7]$ such that $f'(c) = -2$. Since $[5, 7]$ is contained within $[0, 10]$, we may also conclude that the value of c exists on that interval as well. ∎

The last theorem we mention can be viewed as a special case of the Mean Value Theorem.

It isn't really – Rolle's Theorem can be used to prove the MVT, so it is really an important result on its own.

THEOREM 9.3: Rolle's Theorem

Let $f(x)$ be differentiable on (a, b) and continuous on $[a, b]$ where $f(a) = f(b)$. Then there is some point c in (a, b) such that $f'(c) = 0$.

Note that if we have $f(a) = f(b)$ in the MVT, then the average rate of change is 0, and we have the statement of Rolle's Theorem.

Example 9.6: Using Rolle's Theorem

Find the value of c guaranteed to exist by Rolle's Theorem for the function $f(x) = 8\sqrt{x} - x^2 + 3$ on the interval $[0, 4]$.

Solution. The function f is continuous on $[0, 4]$ and differentiable on $(0, 4)$. Note that even though there is point of non-differentiability at $x = 0$, we only require continuity at the endpoint, so this interval is valid. We also have $f(0) = 3$ and $f(4) = 3$. Thus, we may invoke Rolle's Theorem. We have

$$\begin{aligned} f'(x) = 4x^{-1/2} - 2x &= 0 \\ 2x^{-1/2}\left(2 - x^{3/2}\right) &= 0 \\ x^{3/2} &= 2 \\ x &= 2^{2/3} = \sqrt[3]{4}. \end{aligned}$$

Hence, because $0 < \sqrt[3]{4} < 4$, the value of c is $\sqrt[3]{4}$. ∎

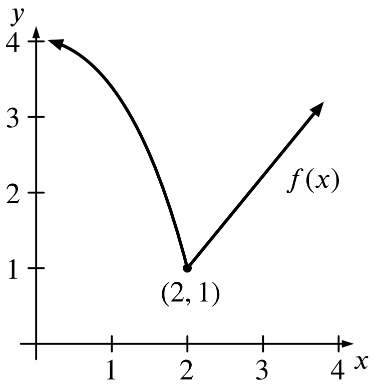
Problems for Lesson 9

You may **not** use a calculator on any of the following problems.

1. Find the value of c that satisfies the Mean Value Theorem for the function $f(x) = x^3$ on the interval $[1, 3]$.

(A) 2 (B) $\frac{\sqrt{39}}{3}$ (C) $\sqrt[3]{13}$ (D) The MVT does not apply.

2. The continuous function f, shown in the figure below, is not differentiable at $x = 2$.



Which of the following are true of f?

I. The Mean Value Theorem applies to f on the interval $[1, 3]$.
II. The Intermediate Value Theorem applies to f on the interval $[1, 3]$.
III. The Extreme Value Theorem applies to f on the interval $[1, 3]$.

(A) III only (B) I and II only (C) II and III only (D) I and III only

3. Suppose g is a continuous function for $-1 \le x \le 4$ and differentiable for $-1 < x < 4$. If $g(-1) = -10$ and $g(4) = 5$, then which of the following must be true of g?

I. $g'(c) = 0$ for some c in the interval $[-1, 4]$.
II. $g'(c) = 3$ for some c in the interval $[-1, 4]$.
III. $g(c) = 3$ for some c in the interval $[-1, 4]$.

(A) I and II only (B) II and III only (C) I and III only (D) II only

4. The function G is continuous on $-1 \le x \le 3$, where $G(-1) = -14$ and $G(3) = 2$. Determine which statement must be true.

(A) $G'(c) = 0$ for some value c in the interval $(-1, 3)$.
(B) $G'(c) = 4$ for some value c in the interval $(-1, 3)$.
(C) $G(c) = 4$ for some value c in the interval $(-1, 3)$.
(D) $-14 \le G(x) \le 2$ for all x in the interval $[-1, 3]$.

5. Let f be the function defined by $f(x) = x^3 + 2x^2 - 4x + k$, where k is a constant. Find the values of k for which f would have exactly two zeros.

(A) -8 and $\frac{40}{27}$ (B) -2 and $-\frac{2}{3}$ (C) -1 and $\frac{8}{9}$ (D) 0 and 4

6. The function g is continuous for $2 \leq x \leq 5$ and differentiable for $2 < x < 5$, where $g(2) = 3$ and $g(5) = -6$. Determine which of the following statements could be false.

I. There exists c for $2 < c < 5$ such that $g'(c) = 0$.
II. There exists c for $2 < c < 5$ such that $g'(c) = -1$.
III. There exists c for $2 < c < 5$ such that $g(c) = 0$.

(A) III only (B) I and II only (C) II and III only (D) I and III only

7. Let f be a function that is continuous for all values and differentiable for $0 < x < 5$ and has values $f(1) = 4$, $f(2) = -2$, and $f(3) = 2$. Which of the following statements could be false?

I. For some c for $1 < c < 3$, $f'(c) = -1$.
II. $f(2)$ is a relative minimum.
III. For some c where $0 < c < 5$, $f(c) = 3$.

(A) I and II only (B) II and III only (C) I and III only (D) II only

8. The function f is continuous on the interval $[0, 2]$ and has values that are given in the table below.

x	0	1	2
$f(x)$	-1	k	-2

Which of the following values of k guarantees that $f(x)$ has at least two zeros in the interval $[0, 2]$?

(A) $-\frac{3}{2}$ (B) $-\frac{1}{2}$ (C) 0 (D) 1

9. Let $f(x) = x^5 - 3x^2 + 4$. Determine the number of values c between $a = -2$ and $b = 2$ for which it is true that

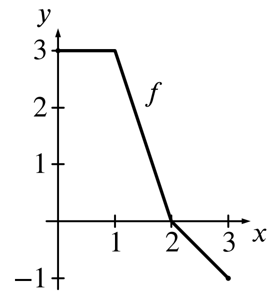
$$\frac{f(b) - f(a)}{b - a} = f'(c).$$

(A) 0 (B) 1 (C) 2 (D) 3

10. Why does the Mean Value Theorem not apply to the function $f(x) = 2(x+1)^{-2}$ on the interval $[-3, 0]$?

(A) f is not defined at $x = -3$ and $x = 0$. (B) $f(-3) \neq f(0)$.
(C) f is not continuous at $x = -1$. (D) Both B and C.

11. The graph of a function f, which consists of three line segments, is shown below.



Suppose g is an antiderivative of f. If $g(3) = 6$, compute $g(0)$.

(A) 1 (B) 2 (C) 3 (D) 4

12. Which of the following is true about the function h defined below?

$$h(x) = \begin{cases} x - 5 & x \leq 2 \\ -3 & 2 < x < 4 \\ 5 - x & x \geq 4 \end{cases}$$

(A) h is continuous at $x = 2$.
(B) The Mean Value Theorem applies to h on the interval $[1, 5]$.
(C) The Intermediate Value Theorem applies to h on the interval $[1, 5]$.
(D) h is continuous at $x = 4$.

13. Which of the following functions satisfy the hypothesis of the Mean Value Theorem on the interval $[0, 3]$?

$$f(x) = \sin(\pi x) + \cos(4x) \qquad g(x) = \sqrt[5]{x-3} \qquad h(x) = \left|x^2 - 3x\right|$$

(A) f and g only (B) f and h only
(C) g and h only (D) f, g, and h

14. For the function $f(x) = \sin(3x)$, how many values of c are guaranteed by Rolle's Theorem on the interval $[0, 2\pi]$?

(A) 0 (B) 2 (C) 4 (D) 6

15. Let $f(x) = x^3 - 12x + 1$ on the closed interval $[-1, 3]$. A line is drawn through the absolute maximum and the absolute minimum on the graph of f on $[-1, 3]$. At what point on the curve is the slope of this line parallel to a tangent line to f?

(A) $(1, -10)$ (B) $\left(\frac{\sqrt{21}}{3}, 1 - \frac{29\sqrt{21}}{9}\right)$

(C) $(2, -15)$ (D) $\left(\frac{\sqrt{57}}{3}, 1 + \frac{7\sqrt{57}}{3}\right)$

16. The number of people in line at the Department of Driver Services is modeled by a twice-differentiable function $P(t)$ for $0 \le t \le 12$. The time $t = 0$ corresponds to 7 AM. The table below shows certain values of $P(t)$.

t (hours)	0	1	2	5	7	10	12
$P(t)$ (people)	25	27	18	45	25	30	35

(a) Using the data in the table, estimate the rate at which the number of people in line was changing at 10:30 AM ($t = 3.5$). Indicate units of measure and show the computations which lead to your answer.

(b) Using a right-hand Riemann sum with subintervals as indicated by the table, estimate $\frac{1}{12}\int_0^{12} P(t)\,dt$. Explain the meaning of this integral in the context of the problem.

(c) For $0 \le t \le 12$, is there a value of t such that $P'(t) = 0$? Justify your answer.

17. The table below shows some values for a twice-differentiable function g.

x	-1	0	1	2	3	4	5
$g(x)$	8	5	-2	1	-1	-3	3

(a) Approximate $\displaystyle\int_{-1}^{5} g(x)\,dx$ using a left Riemann sum over 3 subintervals of equal length.

(b) Using the approximation from part (a), estimate the average value of g over $[-1, 5]$.

(c) Evaluate $\displaystyle\int_{0}^{3} g'(x)\,dx$.

(d) For $-1 < q < 1$, explain why there must exist q such that $g(q) = 0$.

(e) For $-1 < r < 1$, explain why there must exist r such that $g'(r) = -5$.