The AP Calculus Problem Book

Fifth Edition

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The AP Calculus Problem Book

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Set in *STIX*, the Scientific and Technical Information Exchange Font (based on Times New Roman), designed by the STIPub Consortium in 2013, and TEXGyre Adventor, a LATEX implementation of Avant Garde, designed by Herb Lubalin and Tom Carnase in 1970.

This book was produced directly from the author's LATEX files.
Figures were drawn by the author using the TEXdraw package.
Cover image: "Stream of Light 02" by Rajesh Misra.

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Preface

Background

The AP Calculus Problem Book was inspired by Sergio Stadler's "Book of Exercises" at the Marist School in Atlanta. Many of the textbooks in popular use in the early twenty-first century were not written with AP Calculus[1] in mind, and Mr. Stadler wrote a photo-copied, spiral-bound group of worksheets (150 pages!) to supplement his textbook. I was thrilled with the idea, since the textbook I was using—while a very good textbook—was short on AP-type problems and Exam preparation. So I wrote my own "Book." In doing so, I realized that it could be more than just bound worksheets. I could include extra practice problems, practice tests, the syllabus, formula sheets, calculus "discovery" labs, TI-83 calculator instructions, and advice on studying math.

So the first *AP Calculus Problem Book* I wrote in 2002 was similar to the "Book of Exercises": photo-copied and spiral-bound, but had swelled to 250 pages. I copied everything, bought the coils, and bound it using a coil-binding machine the school had purchased. This was made available only to my students, which they purchased for $10 per copy (the money going to the school's Math Team). Over the next few years, I made corrections and tweaks each year, and spent hours and hours each summer assembling the following year's *Problem Book.*

But something unexpected happened. The students liked the *Problem Book* much more than the textbook. So I started teaching from the *Problem Book* more and tailoring my lessons to fit seamlessly with sections of the *Problem Book.* In addition to homework assignments, I started using problems as introductions to material, as "think-pair-share" work, as group work, as options for differentiated instruction, and as ready-made assignments for those days a substitute was needed. By 2007, I stopped using the textbook entirely, and just used the *Problem Book.*

As wonderful as I think the *Problem Book* is, I stopped using it in 2010. In 2007, I started writing an actual textbook, trying to incorporate what I'd learned from using just the *Problem Book* and improving what I found lacking in the popular textbooks. I also grew tired and students grew frustrated with the traditional approach to calculus: the old "precal review, limits, derivatives, integrals" sequence which I felt does not get across the importance or uniqueness of calculus soon enough or adequately enough. So the textbook I wrote[2] is a non-traditional way of approaching calculus, but it also incorporates many of the ideas I learned using the *Problem Book*, as well as some of the problems (and a bunch of new ones).

Why a New Edition?

Over the last fifteen years, I learned a lot about two things: teaching calculus and the mathematical typesetting program, LaTeX. I viewed the old *Problem Book* with the same wincing disdain a great artist might have

[1] I should note here that this book is in no way associated with, endorsed by, or written with the knowledge of anyone at College Board or Educational Testing Service. So there!

[2] "Calculus: Dynamic Mathematics" available on my website `www.drchuckgarner.com` in both AB and BC versions.

for their early work. I decided a few years ago that the *Problem Book* needed updating. What follows are the highlights in the new edition.

- The style of some problems has been updated to match more modern wording.
- The formatting on each page has changed to allow for more problems (there are one thousand more problems in this new edition).
- All the diagrams and graphs were re-drawn.
- The tables have been re-formatted so their layout is consistent.
- There is a second end-of-chapter test to conclude each of the first seven chapters.
- There is an AP-style Scoring Guideline for each free-response problem in each end-of-chapter test.
- There are three new topics in the chapter of post-AP exam problems, to hopefully suit more uses.
- Problems have not just been added to existing pages, but there are entirely new sections of problems added to almost every chapter.
- Missing from the previous editions were problems expressing a definite integral as the limit of a Riemann sum, and vice versa. Such problems are now included.
- More answers to existing problems are included as well as answers to many of the new problems.

I am quite pleased with the new edition, but I'm sure typographical errors persist, and it is quite possible that some of my answers are incorrect. Please contact me if you notice anything in error.

One major change over previous editions is the removal of the TI-83 Calculator Labs. With the ever increasing change in TI calculators and operating systems, the increasing use of computer and web-based graphing software, and the availability of so many TI programs and their instruction on the internet, I felt that keeping the Labs which referenced TI-83s and programs from 13 years ago would be an anachronism. So out they went. However, everything else present in the previous edition is retained in this edition, including retaining Simpson's rule problems even though Simpson's rule is not part of the AP Course Description.[3]

And speaking of the AP Course Description... With the recent (2016-17) change in the AP Course Description, the topic of l'Hôpital's rule is now an AB topic and no longer solely the domain of BC. I thought carefully about moving the section on l'Hôpital's rule out of Chapter 6—a chapter designed for BC only topics—and into Chapter 2 or 3, but I really like the problems in the l'Hôpital's rule section which rely on knowledge of integration. So I left it where it is.[4]

Another change the new Course Description brought with it was the reduction in answer choices for the multiple-choice questions, from five to four. I debated whether to make a corresponding change in the multiple-choice questions in the *Problem Book*, but ultimately decided against it. With answers to the multiple-choice problems in the back of the book, I felt that having five choices rather than four would not make the problems any more difficult.

A Note on Notation

There are two notational aspects of the *Problem Book* that deserve a mention. One is the use parentheses, and the other is the exponential function.

We have all seen the "joke" (or student mistake?) of $\sin x/x = \sin$. Of course the x's do not cancel, and the joke relies on the fact that most of us do not use parentheses consistently to indicate a function's argument. But this is a real problem for students who do not really understand that all of these things—sin, cos, ln, log, and so on—are names of functions. What is really bad is the lack of acknoweldgement that, while students agree that $f(x)$ is a function, $\sin x$ and $\ln x$ somehow are not. In this new edition of the *Problem Book*, I have consistently used parentheses to indicate the argument of a function. I have been following this

[3]Also retained are the interesting, humorous, apropos, questionable, silly, and sometimes puzzling quotes as footnotes, which begin after this dreadfully long preface with the boring unquotable footnotes.

[4]The statement that Chapters 1 through 5 are AB and 1 through 7 are BC is no longer an accurate statement, but it's true except for this one l'Hôpital's rule section.

convention in my classes for years, and it actually does seem to help students realize that these are functions (which does wonders when it comes time for the chain rule or integration by substitution). So there are no instances of "$\sin x$" in this book, only "$\sin(x)$". This is true even of constants, such as $\ln(2)$ or $\arctan(1)$ or $\csc(\pi)$. Notice the similarity with commonly-accepted "$f(4)$" to indicate the evaluation of the function f at the argument 4; then "$\ln(2)$" is the evaluation of the natural logarithm function at the argument 2. I believe this consistency in use of parentheses helps students.

And speaking of functions... The exponential function is one of the most important functions in mathematics. However, when we see "e^x" what conception pops in our heads? Is it a *function*, or is it a *number* raised to a power? I argue that the true importance of the exponential is as a *function*—and we should indicate it as such.

Therefore, in this book I have use a non-italicized e to indicate the name of this function, so that you will not see "e^x" but "e^x". The number 2.71818... is simply the exponential function evaluated at 1: $\mathrm{e}^1 = \mathrm{e}$. I have been using this convention in my classes for years also, and it clears up confusion about what e^x actually is.[5] This also explains why logarithms have a *change-of-base* formula, because exponentials have one too! An exponential function like 3^x is really the exponential *with a change of base:* $3^x = \mathrm{e}^{x\ln(3)}$. The idea that e^x is a number raised to a power is wrong—until the argument of the function takes on a value. Until then, it is as much of a function as $\sin(x)$.

I also prefer the notation "arc" instead of the exponent of -1 to indicate the inverse trigonomatric functions. This reduces confusion with the notation of an exponent of -1 to indicate the reciprocal. So I have avoided using notation such as $\tan^{-1}(x)$ and use exclusively $\arctan(x)$ in this book.

Acknowledgments

Thanks to my students over the years who were forced to solve many of these problems, but especially to the ones who made me make the *Problem Book* better: Megan Villanueva Brown, Raymond Clunie, Mitch Costley, Ashley Jackson Easley, Justin Easley, Carin Godemann Henry, Jonathan Johnson, Nayoon Kim, Adair Kovac, Amy Lanchester, Julie Matthews, Patti Murphy, and James Rives.

Thanks to the Directors of the Rockdale Magnet School for Science and Technology who have supported all of my crazy calculus curriculum ideas: Angela Quick (2001–2005), Mary Ann Suddeth (2005–2014), Debra Arnold (2014–2017), and Amanda Baskett (the current Director).

But big thanks go to my wife, Julie! I spent the better part of the summer of 2017 writing this new edition, and she gave me the time to do so. This is dedicated to her.

Chuck Garner
JULY 2017

Addendum. In January 2021, I finally finished a *Solutions Manual* for all the problems in this book. In doing so, I noticed that there were many answers given in the back which were incorrect, and two problems which were so badly-worded that they were impossible to solve. This is now the "corrected" fifth edition: all the answers have been corrected and the two problems re-written.

[5] It's a function! Indeed, in my classes I sometimes use "$\exp(x)$" to really hammer this in. That may seem strange, but we have two- or three-letter abbreviations for all other common functions (logarithm is log for instance) so why not exp for the exponential?

3.7 Problems of Motion

1122. A car is moving along the highway according to the given equation, where x meters is the directed distance of the car from a given point P at t hours. Find the values of t for which the car is moving to the right and when it is moving to the left. Draw a diagram to describe the motion of the car.

a) $x = 2t^3 + 15t^2 + 36t + 2$

b) $x = 2t^3 + 9t^2 - 60t - 7$

1123. A car is moving along the freeway according to the given equation, where x meters is the directed distance of the car from a given point P at t hours. Find the values of t for which the acceleration is zero, and then find the position of the car at this time.

a) $x = \frac{1}{4}t^4 + \frac{1}{6}t^3 - t^2 + 1$

b) $x = -3\sqrt{t} - \dfrac{1}{12\sqrt{t}}$ for $t > 0$

1124. A snail moves along the x-axis so that at time t its position is given by $x(t) = 3\ln(2t-5)$, for $t > 5/2$.

a) What is the position and the velocity of the snail at time $t = 3$?

b) When is the snail moving to the right, and when is it moving to the left?

1125. An ant moves along the x-axis so that at time t its position is given by $x(t) = 2\cos(\pi t^2/2)$, for values of t in the interval $[-1, 1]$.

a) Find an expression for the velocity of the ant at any given time t.

b) Find an expression for the acceleration at any given time t.

c) Determine the values of t for which the ant is moving to the right. Justify your answer.

d) Determine the values of t for which the ant changes direction. Justify your answer.

1126. A particle is moving along the x-axis so that its position is given by

$$x(t) = \frac{3\pi}{2}t^2 - \sin\left(\frac{3\pi}{2}t^2\right),$$

for $0 < t \leq 2$.

a) Find an expression for the velocity of the particle at any given time t.

b) Find an expression for the acceleration at any given time t.

c) Find the values of t for which the particle is at rest.

d) Find the position of the particle at the time(s) found in part c).

1127. At time $t \geq 0$, the velocity of a body moving along the x-axis is $v(t) = t^2 - 4t + 3$.

a) Find the body's acceleration each time the velocity is zero.

b) When is the body moving forward? Backward?

c) When is the body's velocity increasing? Decreasing?

1128. The position of a ball moving along a straight line is given by $s(t) = \frac{4}{3}e^{3t} - 8t$.

a) Write an expression for the velocity at any given time t.

b) Write an expression for the acceleration at any given time t.

c) Find the values of t for which the ball is at rest.

d) Find the position of the ball at the time(s) found in part c).

3.9 More Tangents and Derivatives

In problems 1155 through 1161, find the tangent lines to each of the following at $x = 0$.

1155. $\sin(x)$

1156. $\cos(x)$

1157. $\tan(x)$

1158. e^x

1159. $\ln(1 + x)$

1160. $(1 + x)^k$, for nonzero constant k.

1161. $(1 - x)^k$, for nonzero constant k.

1162. Using the tangent lines found above, approximate the values of $\sin(0.1)$; $\cos(0.1)$; $\tan(0.1)$; $e^{0.1}$; $\ln(1.1)$; $(1.1)^5$; and $(0.9)^4$.

1163. As noted in problems 1160 and 1161, k is any nonzero constant. Using the tangent found above, approximate $\sqrt{1.06}$; $\sqrt[3]{1.06}$; $1/1.06$; and $1/(1.06)^2$. Then, using your calculator, determine the difference in the approximation compared to the more accurate value given by the calculator.

1164. Let f be a continuous function on $[0, 3]$ that has the following signs and values as in the table below.

x	$f(x)$	$f'(x)$	$f''(x)$
0	0	3	0
$0 < x < 1$	pos.	pos.	neg.
1	2	0	-1
$1 < x < 2$	pos.	neg.	neg.
2	0	dne	dne
$2 < x < 3$	neg.	neg.	neg.
3	-2	-3	0

Find the absolute extrema of f and where they occur; find any points of inflection; and sketch a possible graph of f.

1165. Let $f'(x) = (x - 1)\,e^{-x}$ be the derivative of a function f. What are the critical points of f? On what intervals is f increasing or decreasing? At what points, if any, does f have local extrema?

1166. Let $f'(x) = (x - 1)^2(x - 2)$ be the derivative of a function f. What are the critical points of f? On what intervals is f increasing or decreasing? At what points, if any, does f have local extrema?

1167. A particle moves along the x-axis as described by $x(t) = 3t^2 - 2t^3$. Find the acceleration of the particle at the time when the velocity is a maximum.

1168. Find the values of a, b, c, and d such that the cubic $f(x) = ax^3 + bx^2 + cx + d$ has a relative maximum at $(2, 4)$, a relative minimum at $(4, 2)$, and an inflection point at $(3, 3)$.

1169. Show that the point of inflection of $f(x) = x(x - 6)^2$ lies midway between the relative extrema of f.

1170. Suppose $H(x)$ is a differentiable function whose graph passes through the points $(2, -5)$ and $(5, 4)$. Determine whether each statement must be true or could be false. Explain your reasoning for each statement.

I. $H(x)$ is increasing on the interval $(2, 5)$.
II. The graph of $H(x)$ has x-intercept $(11/3, 0)$.
III. $H'(c) = 0$ for some c in the interval $(-5, 4)$.
IV. $H'(c) = 3$ for some c in the interval $(2, 5)$.
V. $H'(c) = 3$ for all c in the interval $(2, 5)$.

1171. The functions g and h are differentiable for all real numbers. The table below gives values of these two functions and their first derivatives at certain points. The function f is given by $f(x) = 2g(h(x)) - 17$.

x	$g(x)$	$h(x)$	$g'(x)$	$h'(x)$
0	5	1	6	-3
1	8	3	5	-1
2	11	6	-1	1
3	-2	2	-3	4

a) For $0 < c < 3$, must there be a value of c such that $f(c) = 2$? Justify your answer.

b) For $0 < d < 3$, must there be a value of d such that $f'(d) = 2$? Justify your answer.

c) Using values as given in the table, compute $f'(3)$. Show your work.

When introduced at the wrong time or place, good logic may be the worst enemy of good teaching. —*George Polya*

4.3 The Method of Substitution

Find the following indefinite integrals.

1348. $\int -2x\sqrt{9-x^2}\,dx$

1349. $\int x\left(4x^2+3\right)^3\,dx$

1350. $\int \frac{x^2}{\left(1+x^3\right)^2}\,dx$

1351. $\int \left(x^2+\frac{1}{9x^2}\right)\,dx$

1352. $\int \frac{x^2+3x+7}{\sqrt{x}}\,dx$

1353. $\int \left(\frac{t^3}{3}+\frac{1}{4t^2}\right)\,dt$

1354. $\int \sin(2x)\,dx$

1355. $\int \cos(6x)\,dx$

1356. $\int \tan^4(\theta)\sec^2(\theta)\,d\theta$

1357. $\int \frac{\sin(\theta)}{\cos^2(\theta)}\,d\theta$

1358. $\int \cos\left(\frac{\theta}{2}\right)\,d\theta$

1359. $\int x\sqrt{2x+1}\,dx$

1360. $\int x^2\sqrt{1-x}\,dx$

1361. $\int \sqrt{4x-3}\,dx$

1362. $\int x^4\sqrt{3x^5-4}\,dx$

1363. $\int \frac{3x^6}{(2x^7-1)^5}\,dx$

1364. $\int 4x\sqrt{5x-2}\,dx$

1365. $\int 12x^2\sin\left(4x^3\right)\,dx$

1366. $\int 4\,e^x\cos(4\,e^x)\,dx$

1367. $\int 3^{3t}\ln(3)\,dt$

1368. $\int 6^{2x^2-3}x\ln(6)\,dx$

1369. $\int 2^{5x}\,dx$

1370. $\int \frac{1}{\sqrt{5x+4}}\,dx$

1371. $\int 3y\sqrt{7-3y^2}\,dy$

1372. $\int \cos(3z+4)\,dz$

1373. $\int \frac{1}{t^2}\,e^{1/t}\,dt$

1374. $\int \sec\left(x+\frac{\pi}{2}\right)\tan\left(x+\frac{\pi}{2}\right)\,dx$

1375. $\int -\csc^2(\theta)\sqrt{\cot(\theta)}\,d\theta$

1376. $\int \frac{x}{x^2+4}\,dx$

1377. $\int \frac{1}{\sqrt{1-4x^2}}\,dx$

1378. $\int \frac{e^x}{1+e^{2x}}\,dx$

1379. $\int \frac{1}{x}\,dx$

The science of pure mathematics... may claim to be the most original creation of the human spirit. —*Alfred North Whitehead*

4.14 How Do I Find the Area Under Thy Curve? Let Me Count the Ways...

In the following four problems, find the area under the curve on the interval $[a, b]$ by using
a) a right-hand Riemann sum on n equal subintervals;
b) a left-hand Riemann sum on n equal subintervals;
c) 2 trapezoids on equal subintervals;
d) Simpson's rule with 2 parabolas on equal subintervals; and
e) a definite integral.

1549. $y = 2x + 3; \quad [0, 4]; \quad n = 4$

1550. $y = x^2 + 2; \quad [1, 3]; \quad n = 4$

1551. $y = 9 - x^2; \quad [0, 3]; \quad n = 6$

1552. $y = x^3 + 1; \quad [1, 2]; \quad n = 2$

Find the exact area of the region bounded by the given curves.

1553. $y = 16 - x^2, \ y = 0, \ x = 0, \ x = -2$

1554. $y = x^3 + 4, \ y = 0, \ x = 0, \ x = 1$

1555. $y = e^{2x}, \ y = 0, \ x = \ln(2), \ x = \ln(3)$

1556. $y = \tan(x), \ y = 0, \ x = \pi/4$

1557. $y = \dfrac{4}{1 + x^2}, \ y = 0, \ x = 0, \ x = 1$

1558. Determine the area of the region enclosed by the graphs of $f(x) = 9x + 6x^2$ and $g(x) = 3x + 5x^2 + x^3$.

1559. Determine the area of the region enclosed by the graphs of $f(x) = x^2 - 4x$ and $g(x) = x^3 - 6x^2 + 8x$.

Find the average value of each function over the given interval.

1560. $F(x) = 2\sqrt{x - 1}; \ [1, 2]$

1561. $G(x) = e^{-x}; \ [0, 1]$

1562. $J(x) = x^n; \ [1, 2]$ for $n > 1$

1563. $W(x) = 3\cos(3x); \ [0, \pi/6]$

In problems 1564 through 1568, $s(t)$ is position, $v(t)$ is velocity, and $a(t)$ is acceleration. Find both the net distance and the total distance traveled by a particle with the given position, velocity, or acceleration function.

1564. $v(t) = t^2 - 5t + 6$, where $0 \le t \le 3$

1565. $s(t) = 3t^3 - t$, where $0 \le t \le 2$

1566. $a(t) = 2t - 9$, where $0 \le t \le 3$ and $v(2) = 13$

1567. $a(t) = -2t + 1$, where $0 \le t \le 3$ and $v(0) = 0$

1568. $v(t) = e^{\cos(t/2)} \sin(t/2)$, where $0 \le t \le 4\pi$

1569. Particle X moves on the x-axis such that its position at time t, for $0 \le t < \pi/2$, is $x = 1 - \tan(t)$. Particle Y moves on the y-axis with its position given by $y = \sec(t)$, for $0 \le t < \pi/2$. Find the minimum distance that X and Y will be apart.

1570. [Calculator] A sprinter who runs the 100 meter dash in 10.2 seconds accelerates at a constant rate for the first 25 meters and then continues at a constant speed for the rest of the race. Find the sprinter's acceleration.

1571. In the year 2518 AD a spaceship is coming in for a horizontal landing on the moon at 2000 meters per second. The spaceship is to be slowed by an electromagnetic landing track so that during touchdown its velocity v will obey the law $v(t) = 2000 - 20t$ where t is in seconds.

a) At what time t will the spaceship stop?

b) What is the deceleration of the spaceship? Indicate units of measure.

c) What is the total distance covered between touchdown and the spaceship's final stop?

Don't confuse being busy with accomplishment. —*Anonymous*

4.21 AP-Style Integrals Test—Version 1

Section One: Multiple-Choice — No Calculators

Time—30 minutes Number of Questions—15

The scoring for this section is determined by the formula $C \times 1.8$, where C is the number of correct responses. The maximum possible points earned on this section is 27, which represents 50% of the total test score.

Directions: Solve each of the following problems. After examining the form of the choices, decide which is the best of the choices given and fill in the corresponding choice on your answer sheet. Do not spend too much time on any one problem.

Good Luck!

1637. $\displaystyle\int \sin(3\theta)\,d\theta =$

(A) $3\cos(3\theta)+C$ (B) $-3\cos(3\theta)+C$

(C) $-\cos(3\theta)+C$ (D) $\frac{1}{3}\cos(3\theta)+C$

(E) $-\frac{1}{3}\cos(3\theta)+C$

1638. $\displaystyle\int 3^{x^2} x\,dx =$

(A) $\dfrac{3^{x^2+1}}{x^2+1}+C$ (B) $\dfrac{3^{x^2}}{\ln(9)}+C$

(C) $3^{x^2}\ln(3)+C$ (D) $3^{x^3/3}+C$

(E) None of these

1639. $\displaystyle\int_0^5 \frac{dx}{\sqrt{3x+1}} =$

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) 1 (D) 2 (E) 6

1640. The average value of $g(x)=(x-3)^2$ in the interval $[1,3]$ is

(A) 2 (B) $\frac{2}{3}$ (C) $\frac{4}{3}$ (D) $\frac{8}{3}$ (E) $\frac{10}{3}$

1641. If $\displaystyle\int_0^k \frac{\sec^2(x)}{1+\tan(x)}\,dx = \ln(2)$, then the value of k is

(A) $\frac{\pi}{6}$. (B) $\frac{\pi}{4}$. (C) $\frac{\pi}{3}$. (D) $\frac{\pi}{2}$. (E) π.

1642. Which of the following statements are true?

I. If the graph of a function is always concave up, then the left-hand Riemann sums with the same subdivisions over the same interval are always less than the right-hand Riemann sum.
II. If the function f is continuous on the interval $[a,b]$ and $\int_a^b f(x)\,dx = 0$, then f must have at least one zero between a and b.
III. If $f'(x) > 0$ for all x in an interval, then the function f is concave up in that interval.

(A) I only (B) II only (C) III only

(D) II, III only (E) None are true

1643. Let $f(x)$ be defined as below. Evaluate $\int_0^6 f(x)\,dx$.

$$f(x) = \begin{cases} x & 0 < x \le 2 \\ 1 & 2 < x \le 4 \\ x/2 & 4 < x \le 6 \end{cases}$$

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

1644. $\displaystyle\int_0^1 \frac{x}{x^2+1}\,dx =$

(A) $\frac{\pi}{4}$ (B) $\ln\left(\sqrt{2}\right)$ (C) $\frac{1}{2}(\ln(2)-1)$

(D) $\frac{3}{2}$ (E) $\ln(2)$

1645. There is a point between $P(1,0)$ and $Q(\mathrm{e},1)$ on the graph of $y = \ln(x)$ such that the tangent to the graph at that point is parallel to the line through points P and Q. The x-coordinate of this point is

(A) $\mathrm{e}-1$ (B) e (C) -1

(D) $\dfrac{1}{\mathrm{e}-1}$ (E) $\dfrac{1}{\mathrm{e}+1}$

1646. The acceleration of a particle moving along the x-axis at time $t > 0$ is given by $a(t) = 1/t^2$. When $t = 1$ second, the particle is at $x = 2$ and has velocity -1 unit per second. If $x(t)$ is the particle's position, then the position when $t = \mathrm{e}$ seconds is

(A) $x = -2$. (B) $x = -1$. (C) $x = 0$.

(D) $x = 1$. (E) $x = 2$.

1647. If $\int_a^b f(x)\,dx = 3$ and $\int_a^b g(x)\,dx = -2$, then which of the following must be true?

I. $f(x) > g(x)$ for all $a \le x \le b$

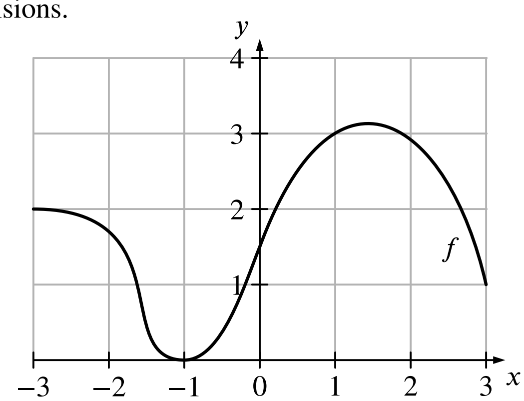
II. $\displaystyle\int_a^b [f(x) + g(x)]\,dx = 1$

III. $\displaystyle\int_a^b [f(x)g(x)]\,dx = -6$

(A) I only (B) II only (C) III only

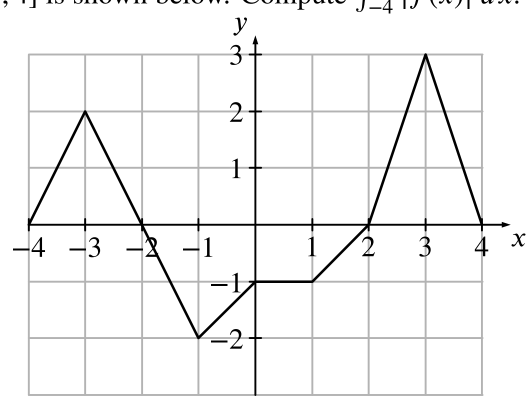
(D) II, III only (E) I, II, and III

1648. The graph of f is shown below. Approximate $\int_{-3}^3 f(x)\,dx$ using the trapezoid rule with 3 equal subdivisions.



(A) $\frac{9}{4}$ (B) $\frac{9}{2}$ (C) 9 (D) 18 (E) 36

1649. The graph of the function f on the interval $[-4,4]$ is shown below. Compute $\int_{-4}^4 |f(x)|\,dx$.



(A) 1 (B) 2 (C) 5 (D) 8 (E) 9

1650. If $f(x) = \displaystyle\int_2^{2x} \frac{1}{\sqrt{t^3+1}}\,dt$, then $f'(1) =$

(A) 0 (B) $\frac{1}{3}$ (C) $\frac{2}{3}$ (D) $\sqrt{2}$ (E) $\frac{\sqrt{3}}{3}$

1651. The area enclosed by the two curves $y = x^2 - 4$ and $y = x - 4$ is given by

(A) $\displaystyle\int_0^1 \left(x - x^2\right)\,dx$ (B) $\displaystyle\int_0^1 \left(x^2 - x\right)\,dx$

(C) $\displaystyle\int_0^2 \left(x - x^2\right)\,dx$ (D) $\displaystyle\int_0^2 \left(x^2 - x\right)\,dx$

(E) $\displaystyle\int_0^4 \left(x^2 - x\right)\,dx$

6.1 A Part, And Yet, Apart...

Find antiderivatives of the following by parts.

1879. $\int x\ln(x)\,dx$

1880. $\int \arctan(x)\,dx$

1881. $\int 2x\,e^{x}\,dx$

1882. $\int 3\theta\sin(2\theta)\,d\theta$

1883. $\int \arcsin(2x)\,dx$

1884. $\int \ln(4x)\,dx$

1885. $\int 2^{x}x\,dx$

1886. $\int \left(x^2-5x\right)e^{x}\,dx$

1887. $\int e^{x}\sin(x)\,dx$

1888. $\int x\sec^2(x)\,dx$

1889. $\int x\sin(x)\,dx$

1890. $\int x^2\sin(x)\,dx$

1891. $\int x^3\sin(x)\,dx$

1892. $\int x\ln\left(\sqrt{x}\right)\,dx$

Solve the differential equations.

1893. $\dfrac{dy}{dx}=x^2e^{4x}$

1894. $\dfrac{dy}{dx}=x^2\ln(x)$

1895. $\dfrac{dy}{d\theta}=\sin\left(\sqrt{\theta}\right)$

1896. $\dfrac{dy}{d\theta}=\theta\sec(\theta)\tan(\theta)$

Solve the following.

1897. Find the area bounded by the curve $y=\ln(x)$ and the lines $y=1$ and $x=e^2$.

1898. Find the area bounded by the curve $y=\ln(x+3)$, the line $y=1$, and the y-axis.

1899. Find the area of the region bounded entirely by the curves $y=\ln(x)$ and $y=\ln^2(x)$.

1900. Find the area between the curves $y=5e^{x}$ and $y=4x^3+\ln(x)$ over the interval $[1,2]$.

1901. Find the volume of the solid generated by revolving the region in the first quadrant bounded by the coordinate axes, the curve $y=e^{x}$, and the line $x=\ln(2)$ about the line $x=\ln(2)$.

1902. Find the average value of $y=2e^{-x}\cos(x)$ over the interval $[0,2\pi]$.

1903. Let R be the region bounded by the graph of $y=\sin(x)$ and the x-axis over the interval $[0,\pi]$. Find the volume of the solid generated when R is revolved about a) the x-axis and b) the y-axis.

1904. Graph the function $f(x)=x\sin(x)$ in the window $0\le x\le 3\pi$, $-5\le y\le 10$, using an x-scale of π and a y-scale of 5. Find the area of the region between f and the x-axis for

a) $0\le x\le\pi$

b) $\pi\le x\le 2\pi$

c) $2\pi\le x\le 3\pi$

d) What pattern do you see here? What is the area between the curve and the x-axis for $n\pi\le x\le(n+1)\pi$ for any nonnegative integer n?

Advertising may be described as the science of arresting human intelligence long enough to get money from it. —*Stephen Leacock*

7.5 More Questions of Convergence...

Which of the series below converge absolutely, which converge conditionally, and which diverge?

2226. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(0.1)^n}{n}$

2227. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n^3+1}$

2228. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{\sin(n)}{n^2}$

2229. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1+n}{n^2}$

2230. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\sqrt[n]{10}$

2231. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n}{n+1}$

2232. $\displaystyle\sum_{n=1}^{\infty}(-5)^{-n}$

2233. $\displaystyle\sum_{n=1}^{\infty}\frac{\cos(n\pi)}{n\sqrt{n}}$

2234. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(n!)^2}{(2n)!}$

2235. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2n)!}{2^n n! n}$

2236. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{\sqrt{n}+\sqrt{n+1}}$

2237. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(1+1/n)^n}$

2238. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{5^n}$

2239. $\displaystyle\sum_{n=1}^{\infty}\frac{(-3)^n}{(n+1)!}$

2240. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(n+2)!}{e^n}$

2241. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{\ln(n)}{n}$

2242. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{n^2}{n+1}$

2243. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{4n}$

2244. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{\sqrt{n}}{n+1}$

2245. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{5n}{n^2+1}$

2246. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n}{n^{1/2}+n^{3/2}}$

2247. $\displaystyle\sum_{n=1}^{\infty}\frac{(-1)^n(n-3)}{(n+1000)^{3/2}}$

In problems 2248 through 2255, estimate the error in using the first four terms to approximate the sum.

2248. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{n}$

2249. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n+1}\frac{1}{10^n}$

2250. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{n!}$

2251. $\displaystyle\sum_{n=1}^{\infty}\left(-\frac{1}{4}\right)^n$

2252. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{n}{100^n}$

2253. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{1}{(2n)!}$

2254. $\displaystyle\sum_{n=2}^{\infty}(-1)^{n}\frac{\ln(n)}{2n}$

2255. $\displaystyle\sum_{n=1}^{\infty}(-1)^{n}\frac{\arctan(n)}{n^2+1}$

2256. Suppose $a_n = \dfrac{1}{\sqrt{n}} + \dfrac{(-1)^n}{n}$ for all positive integers n.

a) Show that $a_n \geq 0$.

b) Show that $\{a_n\} \to 0$ as $n \to \infty$.

c) Explain why the series $\displaystyle\sum_{n=1}^{\infty}(-1)^n a_n$ diverges.

2257. Let $a_n = \dfrac{1+2^n}{1+3(2^n)}$.

a) Does $\displaystyle\lim_{n\to\infty} a_n$ exist? If so, find it.

b) Does $\displaystyle\sum_{n=0}^{\infty} a_n$ converge? Explain.

c) Does $\displaystyle\sum_{n=0}^{\infty}(-1)^n a_n$ converge? Explain.

Even fairly good students, when they have obtained the solution of the problem and written down neatly the argument, shut their books and look for something else. Doing so, they miss an important and instructive phase of the work. ... A good teacher should understand and impress on his students the view that no problem whatever is completely exhausted. One of the first and foremost duties of the teacher is not to give his students the impression that mathematical problems have little connection with each other, and no connection at all with anything else. We have a natural opportunity to investigate the connections of a problem when looking back at its solution. —*George Polya*